WORKING PAPERS

Do the poor benefit less from informal risk-sharing? Risk externalities and moral hazard in decentralized insurance arrangements

Matthieu DELPIERRE[1]
Bertrand VERHEYDEN[1]
Stéphanie WEYNANTS[2]

CEPS/INSTEAD, Luxembourg[1]
University of Namur, Belgium[2]

Working Paper No 2014-08
June 2014

Do the poor benefit less from informal risk-sharing? Risk externalities and moral hazard in decentralized insurance arrangements

Matthieu Delpierre*, Bertrand Verheyden† and Stéphanie Weynants‡§

June 13, 2014

Abstract

Empirical evidence on developing countries highlights that poor farm-households are less keen to adopt high risk / high return technologies than rich households. Yet, they tend to be more vulnerable to income shocks than the rich. This paper develops a model of informal risk-sharing with endogenous risk-taking which provides a rationale for these observations. In our framework, informal risk-sharing is incomplete due to risk externalities, which leads to moral hazard. We compare the first best and second best to a decentralized bargaining process, where the lack of coordination amplifies moral hazard. The analysis of group composition yields counterintuitive results. First, if groups are homogeneous, poor groups share less risks than rich groups even though the rich take more risks. Second, the insurance level of rich households decreases in the presence of poor households, potentially making them reluctant to share risk with poorer households.

JEL CODES: O12, O13

Keywords: Risk-taking, Risk-sharing, Risk externality, Moral hazard

*Marie Curie Fellow at CEPS/INSTEAD, Luxembourg and Earth and Life Institute, Université catholique de Louvain, Belgium; corresponding author, MatthieuDelpierreG@gmail.com

†CEPS/INSTEAD, Luxembourg

‡University of Namur, Belgium

§The authors are grateful to Frédéric Gaspart, Jean-Philippe Platteau, Stefan Dercon, Pierre Dubois, Bruno Henry de Frahan, Tessa Bold, Fabrice Valognes, Anne-Claire Thomas, Philippe Van Kerm, Steve Boucher, Michael Carter and two anonymous referees of the *Journal of Development Economics* for helpful comments and suggestions. The paper has also benefited from presentations at University of Louvain, the CSAE conference in Oxford, the DIAL conference in Paris and the EAAE conference in Dublin.

1 Introduction

In developing countries, the ability of farm households to deal with risk is a key determinant of their daily livelihood as well as their long term economic outcome. In rural regions, formal credit and insurance markets are generally missing. In this context, households try to mitigate the effects of various types of shocks through ex-post coping strategies, individually through buffer stocks such as cattle on the one hand, and collectively through informal insurance transfers on the other hand. However, those ex post strategies generally prove insufficient, forcing households to also adopt ex ante precautionary measures. These measures, which take the form various production plans through crop choices as well as agricultural (traditional versus modern) techniques, imply a trade-off between expected returns and output risks. This trade-off is a potential source of poverty traps for the poorest, who tend to adopt low risk, low return production plans, whereas the rich tend to adopt higher risk, higher return plans. Still, interestingly, the empirical literature has shown that the poor tend to be more affected by idiosyncratic shocks than the rich.[1]

In this paper, we provide a theory of informal risk-sharing with endogenous risk-taking which reproduces the aforementioned stylized facts. Our model takes into account two fundamental specificities of informal risk-sharing arrangements compared to classical insurance models. First, unlike standard insurance markets where shocks can be diluted over a very large number of agents, informal insurance groups are of limited size. Second, informal insurance groups cannot rely on credit to cover a deficit in the event of a bad year for multiple agents, whereas in developed economies, capital markets or markets for reinsurance are available. As a result, informal insurance transfers must adapt to every combination of shocks, in the sense that the insurance group's budget must always be balanced ex post, for all states of the world.[2] A direct consequence of these two features is that agents' risk-taking behavior, which affects the distribution of shocks, generates a negative externality on their partners. More precisely, if a household takes important risks, it is more likely to face large negative shocks, and these shocks need to be covered by its insurance partners. Ex ante, the post-transfer income of these partners is therefore more random. At equilibrium, risk externalities entail moral hazard in the sense that, for a given degree of risk-sharing, risk-taking is higher than the socially optimal level. This moral hazard problem affects the way agents are ready to share risks, and is a potential explanation for the fact that the poor are less well protected against shocks although they take lower risks than the rich. Interestingly, moral hazard is more severe for poor agents, because they have larger returns to risk-taking in terms of expected marginal utility. However, poor agents also are more risk averse and have a stronger willingness to share risks. The net effect of these two opposite forces depends on the composition of the insurance group in terms of wealth or risk aversion. If, as highlighted by most empirical analyses (Ahlin (2010), Giné et al. (2010), Attanasio et al. (2012)), insurance groups are homogeneous, the moral hazard effect dominates the need for insurance among poor groups. In other words, the levels of risk-sharing and risk-taking in poor groups are lower than in richer groups. If the group becomes more heterogeneous, the presence of rich agents who are less reluctant to support the risk externality than the poor should allow the latter to receive more insurance than what they would get in homogeneous groups of identical size. We show that this, however, may not necessarily be the case. In contrast, rich agents always achieve a higher level of risk-sharing in homogeneous groups than in heterogeneous groups of identical size.

[1] References for these claims are provided in the section "Related literature".

[2] Budget balance means here that the sum of interpersonal transfers is equal to zero, whereas in a formal insurance market, this constraint must only hold in expectation.

We first solve the first best problem of the model, in which a social planner designs the risk-sharing scheme and sets agents' levels of risk-taking under the above mentioned constraints imposed by informal insurance. We then compare this constrained first best to two types of risk-sharing arrangements in which agents choose non-cooperatively their risk-taking after observing the insurance scheme. These two types of insurance scheme formation under non-cooperative risk-taking are: (i) the planner's second best solution, which is equivalent to a centralized bargaining problem and (ii) a decentralized bargaining problem in which all potential pairs of agents set a specific transfer scheme so as to maximize the pair's joint surplus. This decentralized bargaining generates group overlaps, which leads to insufficient internalization of risk externalities within pairs since these externalities also hit all other partners. Moral hazard in risk-taking is therefore more problematic in the absence of coordination within the community. In line with empirical findings, both types of arrangements lead to incomplete rates of risk-sharing.

This paper is organized as follows. In Section 2, we provide a survey of the literature. In Section 3 we present the general setting and solve the social planner's first best problem. In sections 4 and 5, agents choose their risk-taking levels non cooperatively after observing the insurance scheme. In Section 4, we study the planner's second best problem. In Section 5, the scheme is instead the result of multiple decentralized bargaining processes between all pairs of agents. In Section 6, we study the impact of insurance groups' composition on risk-sharing schemes. Section 7 concludes.

2 Related literature

The survey is articulated around three fields of the literature which interact in this paper. In the first subsection, we review the literature which treats risk-coping and risk-taking mechanisms, which our models studies simultaneously. In the second subsection, we provide a survey of moral hazard in mutual insurance and explain how endogenous risk-taking relates to this concept. Finally, we cite references linking risk-taking to wealth / risk aversion and risk-sharing.

2.1 Risk-coping and risk-taking

As previously mentioned in the introduction, our paper provides a rationale for the combination of two fundamental stylized facts about risk and poverty in developing countries: (1) poor households tend to be more affected by idiosyncratic risk (Jalan and Ravallion (1999)); (2) poor households take less risk in general (Dercon (1996), Dercon (1998)) and are in particular less keen to adopt high risk / high return technologies (Dercon and Christiaensen (2011)).

Jalan and Ravallion (1999) estimate the fraction of idiosyncratic income shocks that translates into household consumption and find that this fraction tends to be higher for the poor. The inability of the poor to protect themselves against income shocks is due to a limited access to (or use of) risk-coping strategies. These strategies are divided into three important categories.

First, households facing shocks can rely on buffer stocks such as cattle (McPeak (2004), Verpoorten (2009)) or grain (Kazianga and Udry (2006)). However, cattle is also a productive asset and selling it may threaten the household's future livelihood. In that sense, the optimal dynamics of asset accumulation may be incompatible with consumption smoothing when mechanisms of poverty traps are at play. This has been recently attested by Carter and Lybbert (2012) in Burkina Faso. According to them, poor households may engage in asset smoothing rather than consumption smoothing when they feel that their stock of assets is close to a critical threshold, while richer households can afford to use their assets as a buffer.

Second, adjustments in terms of household composition and activities can be made, such as the use of child fostering (Akresh (2009)) and child labor (Jacoby and Skoufias (1997), Beegle et al. (2006), Gubert and Robilliard (2008), Björkman-Nyqvist (2013)).

Third, and central to this article, rural households share risk informally. As already mentioned, this strategy is however known to be imperfect as rural households do not exhaust the gains from sharing their risks, i.e. risk-sharing is incomplete (Townsend (1994); Jalan and Ravallion (1999); Hoogeveen (2002); Murgai et al. (2002)). Development economists have tried to rationalize this phenomenon. Yet, to the best of our knowledge, the classical arguments that are generally invoked in the context of formal insurance markets (i.e. adverse selection and moral hazard) have not been explicitly transposed to the case of informal groups.

Some contributions highlight the lack of contract enforceability (i.e. limited commitment) as a source of incomplete risk-sharing (Kimball (1988), Coate and Ravallion (1993), Kocherlakota (1996), Ligon et al. (2002)).[3] Limited commitment implies that an agent experiencing a favorable outcome relative to his insurance partners and should be in a position to help them has an incentive to renege on this promise. However, the role of wealth and risk aversion as well as the impact of heterogeneity within insurance networks has been overlooked in this literature, with the exception of Coate and Ravallion (1993). They show that informal risk-sharing is potentially more limited in scope for the poor, which is consistent with the first stylized fact we intend to explain. Indeed, the incentive compatibility condition, which restricts the rate of risk-sharing, is binding at lower levels of interpersonal transfers for the poor since their marginal utility of current income is higher. The poor are therefore more reluctant to make transfers when other agents are facing adverse shocks, which explains why they are less protected against income shocks ex post. Our analysis comes to the same conclusion under perfect commitment and endogenous risk-taking, a modeling strategy which allows us to rationalize the two above mentioned stylized facts in a single setting.

Beside limited commitment, the existing theoretical literature has also studied private information as a another source of incomplete risk-sharing. Some papers develop models allowing them to confront predictions on risk-sharing under various types of information imperfections. For instance, Ligon (1998) provides a very general model in which agents are able to hide income and/or actions, and compares the intertemporal pattern of the second best risk-sharing arrangement to the permanent income hypothesis. Karaivanov and Townsend (2014) and Kinnan (2014) consider various regimes including hidden income, moral hazard and limited commitment hypotheses and confront them empirically. However, these papers put very little structure on the moral hazard problem and do not intend to study its consequences on risk-taking, neither do they explore the role of the composition of risk-sharing groups in terms of risk aversion / wealth. Both Ligon (1998) and Kinnan (2014) refer to Rogerson (1985), which studies the role of moral hazard in a dynamic Principal-Agent relationship, as a theoretical basis for the analysis of risk-sharing with moral hazard. Moral hazard in the specific context of informal risk-sharing is however quite distinct from a Principal-Agent relationship, given the interactions between multiple agents and the absence of an explicit authority managing risk-taking in the community. In contrast, we impose more structure on the type of moral hazard and study in detail the consequences of group composition on risk-sharing and risk-taking.

Third, since risk-coping strategies, including buffer stocks and risk-sharing, are imperfect especially for the poor, households also mitigate risks ex ante via risk management strategies (Dercon (2002)). These strategies,

[3] Notice that limited commitment might also be labelled as ex post moral hazard.

which affect the distribution and the magnitude of income shocks, take various forms. First, household may diversify their income sources, both in terms of economic activities (agricultural and non-agricultural sectors) and geographical locations (urban and rural environments) (Morduch (1995), Sarpong and Asuming-Brempong (2004)). Second, households may make use of various crop, production and technological choices. Used efficiently, i.e. on the production frontier, these strategies lead to a trade-off between expected returns and risk. As already mentioned, poor households tend to opt for low risk, low return strategies (Dercon (1996), Dercon (1998), Kurosaki and Fafchamps (2002), Dercon and Christiaensen (2011)).

The following section aims at defining the concept of moral hazard that will be used throughout the paper. It has indeed to be distinguished from the standard formulation of moral hazard in insurance problems. We argue that the pure risk-taking dimension, as opposed to the standard version of moral hazard, is particularly relevant in the context of informal risk-sharing.

2.2 Moral hazard and risk-taking

As previously mentioned, risk management strategies affect the distribution of future income. In classical insurance problems, one generally represents the way in which agents affect the distribution of outcomes in a specific form, which involves an investment in costly actions, or effort. This effort reduces the probability of facing an adverse shock (Arnott and Stiglitz (1988), Arnott and Stiglitz (1991)), and the outcome distribution under high effort is generally considered to *first order stochastically dominate* a low effort distribution.[4] While utility is concave to account for risk aversion, effort costs are generally a separable argument in the utility function.

Instead, we model risk management strategies as a trade-off between the expected output and the variance of income shocks. Agents allocate their resources on the production frontier, so that if they opt for high return strategies, they will face higher risks.[5]

In the context of formal insurance markets, the classical approach (effort to reduce the likelihood of adverse shock) only leads to a moral hazard problem for the insurer if insufficient effort leads to a lower outcome mean. Indeed, if riskier strategies didn't reduce the outcome mean, but were only increasing its variance, insurer profits would remain unchanged on average, while the increase in risk would be handled thanks to the large size of developed economies' markets, the existence of markets for reinsurance and the ability to smooth profits over time via capital markets.

In contrast, in informal insurance groups, moral hazard occurs even if the mean of shocks is unaffected by agents' risk-taking behavior. If the group is of finite size and markets are incomplete, the risk-sharing group's budget constraint has to be satisfied with equality.[6] As previously mentioned, this implies that individual risk-taking affects the (post-transfer) income distributions of all group members. In other words, we show that the context of informal risk-sharing leads to the existence of externalities which are purely related to

[4] Those efforts therefore result in an increase in the outcome mean. However, the impact on the outcome variance may be indeterminate in this setting. Suppose that an agent's income Y is equal to y, with probability $(1-p)$ and $y-s$, with probability p, where $(y,s) \in R_+^2$ and $p \in [0,1]$. Assume that effort e reduces the probability of facing the shock, $p'(e) < 0$. It is easy to see that $\partial E[Y]/\partial e > 0$ and that $\partial Var[Y]/\partial e < 0 \iff p < 1/2$.

[5] Risk management strategies also involve direct costs, as with technology adoption for instance. These costs are considered as implicitely deducted in the final income.

[6] Yet, one can imagine that the group can store resources even when capital markets are absent in order to smooth aggregate income over time. This however depends on the storage technology that is available. If this possibility remains limited, then the budget contraint may bind with a strictly positive probability, which would not affect our main results.

risk.[7] In this sense, moral hazard may occur under the weaker concept of *second order stochastic dominance* (Rothschild and Stiglitz (1970)).

2.3 Wealth, risk aversion and risk-taking

When possibilities to share risk are absent or limited, it is natural to expect that risk-taking will be positively related to risk tolerance, or wealth. The classical theory of entrepreneurship builds on this relationship. Kihlstrom and Laffont (1979) produce a general equilibrium theory of occupational choices. Under missing insurance markets and without any informal possibility to share risk, they show that the identity of entrepreneurs as well as the size of their firms is directly explained by wealth when preferences are characterized by decreasing absolute risk aversion. More recently, Newman (2007) falsified this prediction by adding the possibility of risk-sharing. In this paper, he shows that a setting based on wealth heterogeneity, endogenous risk-taking and risk-sharing with moral hazard may lead to implausible predictions, namely that the poor become the entrepreneurs. Newman (2007)'s model differs from our approach in several ways. First, it adopts the standard approach to moral hazard described in the previous subsection (first order stochastic dominance induced by a costly effort). Second, Newman (2007)'s setting considers risk-sharing with a continuum of agents, which prevents the problem of the imperfect diversification of risks that is inherent to informal risk-sharing groups motivated in our paper. These differences lead us to divergent conclusions. Indeed, he finds that moral hazard is more severe for the rich in the sense that, to produce the incentive compatible level of effort, they need to bear more risk. In other words, moral hazard in the classical approach leads the rich to receive a lower insurance coverage. In contrast, we find that moral hazard mainly prevents the poor from sharing risk efficiently. As developed below, the reason is that the poor are more sensitive to a marginal increase in their insurance coverage, so that their response in terms of risk-taking generates more negative externalities. The reason for this opposition in predictions is therefore due to the absence of risk externalities in Newman's setting.

Fischer (2013), which examines risky investments and risk-sharing within microfinance groups, also shares similarities with our paper. He compares the performance of alternative contractual forms, such as individual liability, joint liability, and equity contracts, and shows that joint liability, which fosters peer monitoring, may hamper risky investments, thereby reducing the profitability of the economic activities financed by micro-loans. As in our model, he finds that agents who are more risk tolerant / richer may engage more in risk-sharing. The mechanisms behind this similarity are however different. In Fischer (2013), risk tolerant agents tend to invest more in the risky assets and therefore have more risk to share than risk averse agents. In our model, the presence of risk externalities plays again a crucial role to explain this result. Moral hazard in risk-taking leads to lower insurance coverage. Since, as previously mentioned, moral hazard is more prevalent among poor, these agents end up being less insured than the rich.

[7] Note that in our model, different risk-taking strategies imply differences in shock variance as well as differences in the income mean. These differences in mean affect the lump sum component of interpersonal transfers, but the unique source of inefficiency in risk-sharing stems from pure risk externalities.

3 The general model

3.1 Technology and preferences

Let us consider a set $H = \{1, ..., n\}$ of n households which may engage in risk-sharing. Income y_h is random and its distribution is affected by the household's risk management choices embodied by the decision variable $\sigma_h \in [0; \overline{\sigma}]$, which captures the level of risk taken by household h. We assume that the first and second moments of y_h are affected by σ_h in the following way:

$$\begin{aligned} E(y_h; \sigma_h) &= \mu(\sigma_h), \\ Var(y_h; \sigma_h) &= \sigma_h^2, \end{aligned}$$

where $\mu'(\sigma) > 0$ and $\mu''(\sigma) < 0$ for all $\sigma \leq \overline{\sigma}$.[8] The fact that σ_h increases both the mean and the variance of y_h implies a trade-off between risk and expected return. One can interpret this representation as the production frontier of the set of technologies available to households.[9,10] For expositional simplicity, one can rewrite income as

$$y_h = \mu(\sigma_h) + s_h, \tag{1}$$

where $s_h \in \mathbb{R}$ is a random shock of mean $E(s_h; \sigma_h) = 0$ and variance $Var(s_h; \sigma_h) = \sigma_h^2$. Shocks are independent between households: $s_i \perp s_j$, for all $i \neq j$ in H.[11] A state of the world S is a specific realization of all households' income shocks: $S = (s_1, ..., s_n)' \in \mathbb{R}^n$. Households can protect themselves against these shocks via an informal risk-sharing arrangement within the group. In this arrangement, they commit to make reciprocal income-contingent transfers.[12] As argued in the introduction, informal insurance groups cannot rely on credit to cover potential deficits. A direct consequence of these characteristics is that the transfer scheme must be budget-balanced for all possible states of the world. This implies that, contrary to classical insurance problems, each household's transfer is a function of all the shocks faced by all households: $t_h = t_h(S)$, and budget balance imposes that for all S,

$$\sum_{h \in H} t_h(S) = 0.$$

The vector of income transfers received by each household is denoted by $T = (t_1, ..., t_n)' \in \mathbb{R}^n$. A risk-sharing arrangement maps a vector of shocks S into a vector of transfers T in the following way:

$$T = L + \Gamma' S, \tag{2}$$

where $L = (l_1, ..., l_n)' \in \mathbb{R}^n$ is a vector of lump sum transfers, i.e. transfers that are independent of S, and

[8] In order to avoid third order effects in some comparative statics, we also assume that $\mu'''(\sigma) = 0$.

[9] Any technological choice below this frontier would be inefficient as, from this point, it would be possible to strictly increase the expected income without increasing risk.

[10] All households have access to the same set of technologies, but may opt for different strategies at equilibrium.

[11] Notice that a covariate shock could easily be added. It would simply imply that some fraction of the variance cannot be reduced by risk-sharing at the group level. However, since one of our objectives is to describe pure risk externalities, ignoring covariate shocks reinforces our point. Indeed, we show that individual risk-taking generates externalities even if shocks are independent.

[12] In order to describe the mechanism behind risk externalities in the clearest way, we assume that these transfers are enforceable. In this way, the implications of moral hazard in risk-taking are clearly distinct from the limited commitment argument.

where the $(n \times n)$ matrix Γ determines how the realization of shocks in the group affect all members' transfers:

$$\underset{(n\times n)}{\Gamma} = \begin{pmatrix} -\alpha_1 & \gamma_{12} & \cdots & \cdots & \gamma_{1n} \\ \gamma_{21} & -\alpha_2 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & -\alpha_{(n-1)} & \gamma_{(n-1)n} \\ \gamma_{n1} & \cdots & \cdots & \gamma_{n(n-1)} & -\alpha_n \end{pmatrix}.$$

The diagonal element $\Gamma_{h,h} = -\alpha_h$ represents the share of household h's shock which is insured by the group, while the off-diagonal element γ_{jh} represents the share of household j's shock that household h commits to cover. The budget constraint reduces the scheme's total number of parameters by imposing the following structure.

Lemma 1 *The risk-sharing arrangement's budget constraint imposes that*

1. $\sum_{h\in H} l_h = 0$,
2. *for all* $h \in H$, $\alpha_h = \sum_{i\in H\backslash\{h\}} \gamma_{hi}$.

Proof. Provided in Appendix 1. ■

For the budget constraint to be satisfied, two conditions need to be met. On the one hand, the sum of lump sum transfers should be zero, otherwise the group would generate a surplus or a deficit on average. In particular, it can be easily seen that in the state of the world where $S = (0, ..., 0)'$, the budget constraint would be violated. On the other hand, the insured fraction of household h's shock, α_h, is equal to the sum of the fractions of h's shock that the other members commit to cover, $\sum_{i\in H\backslash\{h\}} \gamma_{hi}$. It is worth noting that the informal insurance's budget constraint makes full insurance impossible since under full insurance, the risk sharing arrangement would be such that $\Gamma = -I_n$, where I_n is a $(n \times n)$ identity matrix.

Making use of (2), we can write the transfer received by household h as

$$t_h (S; L, \Gamma) = l_h - \alpha_h s_h + \sum_{i\in H\backslash\{h\}} \gamma_{ih} s_i. \tag{3}$$

The consumption level of household h after transfer is obtained by combining (1) and (3):

$$\begin{aligned} c_h (S; L, \Gamma) &= k_h + y_h + t_h \\ &= k_h + \mu (\sigma_h) + l_h + (1 - \alpha_h) s_h + \sum_{i\in H\backslash\{h\}} \gamma_{ih} s_i, \end{aligned} \tag{4}$$

where k_h is household h's wealth. The mean and variance of consumption are

$$E (c_h; \sigma_h, L) = k_h + \mu (\sigma_h) + l_h, \tag{5}$$

$$Var (c_h; \Gamma, \Sigma) = (1 - \alpha_h)^2 \sigma_h^2 + \sum_{i\in H\backslash\{h\}} \gamma_{ih}^2 \sigma_i^2, \tag{6}$$

by independence between s_i and s_j, for all $i \neq j$ in H. The consumption equation (4) shows that informal risk-sharing may allow a household h to reduce its exposure to its own income shock by α_h. This fraction of the shock is supported by the other group members since by Lemma 1, $\alpha_h = \sum_{i \in H \backslash \{h\}} \gamma_{hi}$. The fact that shocks are passed to other members affects these members' utilities through their consumption variance (see 6). This implies that risk-sharing is a source of risk externalities, since households nolonger fully internalize the adverse effects of their risk-taking behavior, which now affects other members. This will be the case as soon as off-diagonal elements of Γ are different from zero (i.e. $\gamma_{ij} \neq 0$), which is imposed by any insurance scheme's budget constraint.[13]

Informal risk-sharing allows household h to reduce its own shock variance to $(1-\alpha_h)^2 \sigma_h^2$ (see equation 6), while other members, who support part of this risk, increase their variance by $\gamma_{hi}^2 \sigma_h^2$. The interest of sharing risk is that the aggregate impact of σ_h^2, $\left[\left(1-\sum_{i \in H \backslash \{h\}} \gamma_{hi}\right)^2 + \sum_{i \in H \backslash \{h\}} \gamma_{hi}^2\right] \sigma_h^2$, is always smaller than the risk in autarky, σ_h^2. To illustrate this, let us consider the homogeneous case where $\alpha_i = \alpha$ and $\gamma_{hi} = \frac{\alpha}{n-1}$ for all i. The aggregate impact in the group of σ_h^2 then boils down to $1 - \alpha\left(2 - \frac{n}{n-1}\alpha\right)$, which is equal to 0 (i.e. risks are fully diversified) when n tends to infinity and α tends to 1. In other words, when the group is of infinite size, full risk-sharing ($\alpha \to 1$, $\gamma \to 0$) allows perfect diversification and makes risk externalities disappear. Summing up, when capital and insurance markets are missing and informal risk-sharing groups are of finite size, individual risk-taking generates risk externalities. These externalities will generate moral hazard problems when actions are imperfectly monitorable.

Agents are risk averse and derive utility from consumption. The utility function of a household h, $u_h(c)$ is therefore such that $u' > 0$ and $u'' < 0$, with constant absolute risk aversion a_h.[14,15] The vector of household risk aversions is noted $A = (a_1, ..., a_n) \in \mathbb{R}_+^n$. The certainty equivalent $\tilde{c}_h$ is by definition such that

$$u(\tilde{c}_h) = E_S\left[u\left(c_h\left(t_h(S; L, \Gamma)\right)\right)\right] = \int \cdots \int u\left(c_h\left(t_h(S; L, \Gamma)\right)\right) f(s_1, ...s_n)\, ds_1 ... ds_n, \tag{7}$$

where $f(s_1, ...s_n)$ is the joint density of shocks in the group. Making use of Pratt's approximation of the risk premium, we can write

$$\tilde{c}_h \approx E(c_h; \sigma_h) - \frac{a_h}{2} Var(c_h; \Gamma, \Sigma) = k_h + \mu(\sigma_h) + l_h - \frac{a_h}{2}\left((1-\alpha_h)^2 \sigma_h^2 + \sum_{i \in H \backslash \{h\}} \gamma_{ih}^2 \sigma_i^2\right). \tag{8}$$

3.2 The first best allocation

Under the above mentioned technological and institutional constraints imposed on rural risk-sharing groups, we first consider the first best problem of a social planner who designs the risk-sharing scheme *and* is able to enforce households' risk-taking σ_h. This planner seeks to maximize a social welfare function W aggregating the expected utilities of all households in the group, with respect to the vector of lump sum transfers L, the risk-sharing arrangement Γ, and the risk-taking profile Σ:

$$\underset{L, \Gamma, \Sigma}{Max} W = \sum_{h \in H} \lambda_h u(\tilde{c}_h), \tag{9}$$

[13] One could argue that risks are also shared with agents outside the community, such as migrants, allowing the group to survive to structural losses. However, as soon as the risk cannot be fully diversified, a residual risk remains at the group level, which is the mechanism on which we concentrate.

[14] In what follows, we will simplify notations by ignoring the subscript on the utility function.

[15] In our framework, households are heterogeneous in wealth and risk aversion, richer households having a lower level of risk aversion. This representation of utility can be seen as a case of decreasing absolute risk aversion across agents. However, from an individual's viewpoint, local variations in income do not affect affect risk aversion. It must also be noted that, while we are able to determine the shape of the risk-sharing scheme under wealth heterogeneity, this heterogeneity per se does not play a crucial role as it is managed by the lump sum transfer. Heterogeneity in risk aversion is instead crucial.

where λ_h is the Pareto weight attributed to household h. Let us denote by $\tau_h = \frac{1}{a_h}$ the risk tolerance of household h.

Proposition 1 *At the first best,*

1. *the vector of lump sum transfers L^{FB} is such that for all $i, j \in H$,*

$$\frac{u'(\tilde{c}_i)}{u'(\tilde{c}_j)} = \frac{\lambda_j}{\lambda_i}, \tag{10}$$

2. *the risk-sharing arrangement Γ^{FB} is such that for all $i, j \in H$,*

$$\begin{aligned} \gamma_{ij}^{FB} &= \frac{\tau_j}{\sum\limits_{h \in H} \tau_h}, \\ \alpha_i^{FB} &= 1 - \frac{\tau_i}{\sum\limits_{h \in H} \tau_h}, \end{aligned} \tag{11}$$

3. *risk-taking is homogeneous: $\Sigma^{FB} = \left(\sigma^{FB}, ..., \sigma^{FB}\right)$, where σ^{FB} is such that*

$$\mu'\left(\sigma^{FB}\right) = \frac{\sigma^{FB}}{\sum\limits_{h \in H} \tau_h}. \tag{12}$$

Proof. Provided in Appendix 2. ■

At the first best, the Pareto frontier is defined by the socially optimal levels of risk-sharing Γ^{FB} and risk-taking Σ^{FB}. Based on the Pareto weights $\Lambda = (\lambda_1, ..., \lambda_n)$, the role of the vector of lump sum transfers L^{FB} is to select a particular point on this frontier by equalizing the ratios of marginal utilities to the ratios of pareto weights (10).[16]

The first best risk-sharing arrangement Γ^{FB} (11) imposes that individual income shocks s_h are shared across households according to their relative level of risk tolerance: $\gamma_{ij}^{FB} = \frac{\tau_j}{\sum\limits_{h \in H} \tau_h}$. More risk-tolerant households also bear more of their own shock, since $1 - \alpha_j^{FB} = \frac{\tau_j}{\sum\limits_{h \in H} \tau_h}$. Therefore, the relative level of risk tolerance determines the way in which the total sum of income shocks, $\sum\limits_{i \in H} s_i$, is shared between households, since

$$c_h\left(S; \Gamma^{FB}, \Sigma^{FB}\right) = k_h + l_h^{FB} + \mu\left(\sigma_h^{FB}\right) + \frac{\tau_h}{\sum\limits_{i \in H} \tau_i} \sum_{i \in H} s_i. \tag{13}$$

As regards risk-taking Σ^{FB}, two things are worth mentioning.

First, despite potential differences in risk aversion, all agents take the same level of risk at the first best. This is due to the fact that the planner is able to reallocate consumption among households and at the same time to prevent them from adjusting their risk-taking behavior. This does not hold anymore in the second best, where Σ is not enforceable. Equation (12) states that the (unique) optimal risk-taking level is such

[16] At this stage, we are agnostic about Λ. If pareto weights are equal across agents, certainty equivalents must be equal for all agents at the first best.

that its *social* marginal benefit μ' (i.e. the increase in total expected output by an individual) exactly offsets its marginal *social* cost $\sigma^{FB}/\sum_{i\in H}\tau_i$, which represents the increase in the *social* risk premium. These social costs and benefits can be obtained by taking the certainty equivalent of c_h (13) and by aggregating it over all individuals:

$$\sum_{h\in H}\tilde{c}_h^{FB}=\sum_{h\in H}k_h+\sum_{h\in H}l_h^{FB}+\sum_{h\in H}\mu\left(\sigma_h^{FB}\right)-\frac{1}{2}\frac{1}{\sum_{i\in H}\tau_i}\sum_{i\in H}\sigma_i^2.$$ [17]

Second, the socially optimal risk-taking level is determined by the willingness to bear risk at the group level, namely the aggregate risk tolerance $\sum_{h\in H}\tau_h$. One can indeed see from (12) that the first best level of risk-taking σ^{FB} is increasing in aggregate risk tolerance $\sum_{h\in H}\tau_h$. This implies that the socially optimal level of risk-taking increases mechanically with the size of the insurance group, which is due to a better diversification of risks.

4 Second best analysis: risk-sharing with moral hazard

Let us now study the case in which the social planner is not able to enforce households' risk-taking behavior. The transfer scheme (L,Γ) maps S into T and accounts for households' levels of risk aversion A. As we will see, the insurance scheme affects individuals' risk-taking Σ, and even though Σ is not enforceable, observing A allows the planner to anticipate the impact of (L,Γ) on Σ. In this second best analysis, the planner's problem is therefore to maximize the social welfare function by setting the insurance scheme (L,Γ), anticipating the scheme's impact on individuals' non-cooperative risk-taking Σ.

We start by solving the household's individual risk-taking problem, considering the risk-sharing arrangement as given. The following Lemma characterizes the equilibrium non-cooperative risk-taking profile Σ^N.

Lemma 2 ***Non-cooperative risk-taking*** *The equilibrium non-cooperative risk-taking profile* $\Sigma^N(\Gamma)=\left(\sigma_1^N(\alpha_1),...,\sigma_n^N(\alpha_n)\right)$ *is such that*

$$\mu'\left(\sigma_h^N\right)=a_h\left(1-\alpha_h\right)^2\sigma_h^N. \tag{14}$$

Proof. Considering the transfer scheme as given, households maximize expected utility $u(\tilde{c}_h)=E_S\left[u\left(c_h\left(t_h\left(S;L,\Gamma\right)\right)\right)\right]$, where $\tilde{c}_h$ is determined by equation (8). σ_h^N is obtained by taking the first order condition of the household's utility maximization problem with respect to σ_h.[18] ■

Let us comment the positive and normative aspects of σ_h^N. First, risk-taking decreases with risk aversion, $\partial\sigma_h^N/\partial a_h<0$. Therefore, as soon as group members have different degrees of risk aversion, risk-taking is not homogeneous as in the first best. Also, household h's risk-taking increases with its own rate of insurance coverage: $\partial\sigma_h^N/\partial\alpha_h>0$: the larger the share of its shock that a household can externalize through risk-sharing, the higher the level of risk it takes. Let us define ϵ_h as the elasticity of non-cooperative risk-sharing to the rate of coverage α_h, a concept that will be used later on to highlight the impact of moral hazard on risk-sharing:

$$\epsilon_h=\frac{\partial\sigma_h^N}{\partial\alpha_h}\frac{\alpha_h}{\sigma_h^N}=\frac{2\left(1-\alpha_h\right)\alpha_h}{-\mu''\left(\sigma_h^N\right)\tau_h+\left(1-\alpha_h\right)^2}>0. \tag{15}$$ [19]

[17] This can be done because, by the lump sum transfers, we know that $\lambda_h u'(c_h)$ is identical for all h.

[18] As the strategy of the other households do not appear in household h's reaction function, we have an equilibrium in dominant strategies.

[19] This formula is obtained by applying the implicit function theorem to equation (14).

Second, as regards the normative aspects of σ_h^N, the private marginal benefit of non-cooperative risk-taking, $\mu'\left(\sigma_h^N\right)$, is equal to its private marginal cost, which by definition does not incorporate the risk externalities generated by σ_h^N on other members (i.e. the shares of h's risk that are borne by its insurance partners). As a result, σ_h^N is too high compared to the social optimum, leading to moral hazard as stated in the next proposition.

Proposition 2 ***Moral hazard*** *Non-cooperative risk-taking is always larger than the socially optimal level under the same arrangement* Γ*:*

$$\sigma_h^N(\Gamma) > \sigma_h^{FB}(\Gamma),$$

where $\sigma_h^{FB}(\Gamma)$ *is the social planner's first best level of risk-taking for any given* Γ*.*

Proof. As in Appendix 2, $\sigma_h^{FB}(\Gamma)$ is obtained by the planner's first order condition with respect to σ_h, with the only distinction that Γ is here left undefined so as to highlight the fact that the moral hazard phenomenon is general to any insurance scheme. This first order condition states that

$$\frac{\partial W}{\partial \sigma_h} = \lambda_h u'(\tilde{c}_h)\frac{\partial \tilde{c}_h}{\partial \sigma_h} + \sum_{i \in H\backslash\{h\}} \lambda_i u'(\tilde{c}_i)\frac{\partial \tilde{c}_i}{\partial \sigma_h} = 0, \tag{16}$$

which, using (10) and taking the derivative of $\tilde{c}_h$ and $\tilde{c}_i$ with respect to σ_h in (8), leads to:

$$\frac{\mu'\left(\sigma_h^{FB}(\Gamma)\right)}{\sigma_h^{FB}(\Gamma)} = a_h\left(1-\alpha_h\right)^2 + \sum_{i \in H\backslash\{h\}} a_i \gamma_{hi}^2.$$

Comparing this expression to (14), one can see that for any Γ, $\frac{\mu'\left(\sigma_h^{FB}(\Gamma)\right)}{\sigma_h^{FB}(\Gamma)} > \frac{\mu'\left(\sigma_h^{N}(\Gamma)\right)}{\sigma_h^{N}(\Gamma)} = a_h\left(1-\alpha_h\right)^2$. Since $\frac{\mu'(\sigma)}{\sigma}$ is decreasing in σ, we can conclude that $\sigma_h^{FB}(\Gamma) < \sigma_h^N(\Gamma)$ for all Γ. ■

This proposition states that non-cooperative risk-taking is always higher than the risk-taking level that would have been chosen by the social planner, had this risk-taking been enforceable. The reason thereof is that, while the household only considers its private marginal cost of risk-taking $\lambda_h u'(\tilde{c}_h)\frac{\partial \tilde{c}_h}{\partial \sigma_h}$, the social planner also takes into account the externalities generated by risk-taking on the other group members, as highlighted in (16) by $\sum_{i \in H\backslash\{h\}} \lambda_i u'(\tilde{c}_i)\,\partial\tilde{c}_i/\partial\sigma_h$.

This result may appear at odds with empirical observations of risk-taking behaviors in developing countries, which highlight that risk-taking is limited (see, for instance Kurosaki and Fafchamps (2002)). However, since risk-taking is generally excessive, risk-sharing arrangements are adjusted to mitigate the moral hazard problem, which leads to limited insurance. As a result, under the second best risk-sharing arrangement, it may well be that $\sigma_h^N\left(\Gamma^{SB}\right) < \sigma^{FB}$, as shown in Proposition 5. This proposition indeed consists in describing the design of the transfer scheme $\left(L^{SB}, \Gamma^{SB}\right)$ set by the planner in the first stage of the game (i.e. before individual risk-taking is chosen), anticipating that households will adopt the non-cooperative level of risk-taking $\Sigma^N(\Gamma)$.

Proposition 3 *At the second best,*

1. *the vector of lump sum transfers* L^{SB} *is such that for all* $i, j \in H$*,*

$$\frac{u'(\tilde{c}_i)}{u'(\tilde{c}_j)} = \frac{\lambda_j}{\lambda_i},$$

2. *the risk-sharing arrangement* Γ^{SB} *is such that for all* $i,j \in H$,

$$\gamma_{ij}^{SB} = \frac{\tau_j}{\sum_{h\in H} \tau_h + \tau_i \epsilon_i},$$
$$\alpha_i^{SB} = 1 - \frac{\tau_i (1+\epsilon_i)}{\sum_{h\in H} \tau_h + \tau_i \epsilon_i}. \tag{17}$$

3. *the risk-taking profile is* $\Sigma^{SB} = \Sigma^N(\Gamma^{SB}) = (\sigma_1^N(\alpha_1^{SB}), ..., \sigma_n^N(\alpha_n^{SB}))$, *where* $\sigma_i^{SB} = \sigma_i^N(\alpha_i^{SB})$ *is given by*

$$\mu'(\sigma_i^{SB}) = \tau_i \left(\frac{1+\epsilon_i}{\sum_{h\in H} \tau_h + \tau_i \epsilon_i} \right)^2 \sigma_i^{SB}. \tag{18}$$

Proof. Provided in Appendix 3. ■

Proposition 3 describes the equilibrium of the game. In order to provide an interpretation of this equilibrium, we compare it to the first best in terms of risk-sharing and risk-taking.

Proposition 4 *The second best level of risk-sharing is lower than the first best level:*

$$\gamma_{ij}^{SB} = \Phi_i^{SB} \gamma_{ij}^{FB} < \gamma_{ij}^{FB}, \tag{19}$$
$$\alpha_i^{SB} = \Phi_i^{SB} \alpha_i^{FB} < \alpha_i^{FB}, \tag{20}$$

where

$$\Phi_i^{SB} = \frac{\sum_{h\in H} \tau_h}{\sum_{h\in H} \tau_h + \tau_i \epsilon_i} \in (0,1). \tag{21}$$

Proof. This result is derived from a direct comparison of equations (11) and (19). ■

The fraction of shock γ_{ij} that household i transfers to household j is lower for all $i,j \in H$ under the second best. Conversely, i's coverage α_i^{SB} is lower than the first best level (20). One can immediately see that the difference between the first best and second best levels of risk-sharing is attributable to the presence of moral hazard, captured by the term $\epsilon_i \equiv \frac{\partial \sigma_i^N}{\partial \alpha_i} \frac{\alpha_i}{\sigma_i^N}$ in (21). Indeed, if risk-taking was enforceable by the planner and households were not able to adapt their risk-taking behavior to the insurance scheme (i.e. $\epsilon_i = 0$), then Φ_i^{SB} would be equal to 1, and the second best risk-sharing arrangement would be identical to the first best. The main intuition behind this result is that, in the second best, the planner faces a trade-off between risk-sharing and incentives: as seen above, risk-sharing leads to higher risk-taking, so that risk-sharing has to be reduced compared to the first best in order to temper moral hazard.

Another comment pertains to the way shocks are shared between agents. We noticed at the first best that a household supports the same fraction of all households' shocks, including its own ($1 - \alpha_h^{FB} = \gamma_{ih}^{FB}$). In order to temper moral hazard under the second best, households must support a larger fraction of their own shock than the fraction that they support from their partners ($1 - \alpha_h^{SB} > \gamma_{ih}^{SB}$).[20]

[20] Indeed, $1 - \alpha_h^{SB} > \gamma_{ij}^{SB}$ if and only if $1 - \Phi_i^{SB}\alpha_i^{FB} > \Phi_i^{SB}\gamma_{ij}^{FB}$, or equivalently $1 > \Phi_i^{SB}\left(\gamma_{ij}^{FB} + \alpha_i^{FB}\right) = \Phi_i^{SB}$, which is always the case.

Finally, it is also worth highlighting that the reduction in insurance coverage is proportionally larger for households more subject to moral hazard, that is, households characterized by a higher elasticity of risk-taking to risk-sharing. This means that households more prone to taking risks when they are insured receive less insurance under the second best, other things equal. Interestingly, we will show below that those are the households with the highest levels of risk aversion.

The second comparison that deserves attention pertains to risk-taking.

Proposition 5 *The second best level of risk-taking is lower than the first best level ($\sigma_h^{SB} < \sigma_h^{FB}$) if and only if*

$$\epsilon_h > \sqrt{\frac{1}{\tau_h} \sum_{i \in H} \tau_i}.$$

Proof. Provided in Appendix 3. ■

Two conflicting effects are at play here. On the one hand, the moral hazard problem leads households to take too much risk for the level of insurance that they receive. On the other hand, anticipating that effect, the planner tempers households' risk-taking behavior by offering a lower rate of risk-sharing (than at the first best). Proposition 5 tells us that if the moral hazard problem is important, i.e. if ϵ_h is too large, then the reduction in α_h^{SB} is so strong that risk-taking is lower that the first-best level.

5 Decentralized risk-sharing arrangements

The second best approach is standard in the risk-sharing literature. However, implementing the second best risk-sharing arrangement Γ^{SB} described above requires a high degree of coordination at the group level. It can indeed easily be shown that the second best approach encompasses the outcome of a centralized bargaining in the whole community. This outcome may however not be sustainable as soon as, for instance, households are able to deviate from the centralized risk-sharing scheme by negotiating bilateral agreements.[21] We therefore study in this section the arguably more realistic case in which households bargain over risks-sharing arrangements in a decentralized way. Under decentralized bargaining, all potential pairs of households set a specific transfer scheme so as to maximize the pair's joint surplus.

Let us start by formalizing the bilateral bargaining process. Each pair of households $\{i, j\} \subset H$ negotiates a risk-sharing contract $(l_{ji}, \gamma_{ij}, \gamma_{ji})$ which determines the transfers that they commit to make to each other after the realization of income shocks. Consistently with our previous definitions, the net transfer given by j to i writes

$$t_{ji} = -t_{ij} = l_{ji} - \gamma_{ij} s_i + \gamma_{ji} s_j,$$

where $l_{ji} = -l_{ij}$ is the lump sum transfer going from j to i. Notice that the budget constraint is automatically satisfied within a pair as $t_{ji} = -t_{ij}$, or equivalently $t_{ji} + t_{ij} = 0$. We assume that the other bilateral agreements that i and j have concluded with other households in H are taken as given when they negotiate. Still, these agreements with third parties matter as they define their respective exit options. Let us therefore define $\bar{u}_{i,-j}$ as the expected utility of household i if it had bilateral links with all households but j. Summing up, for all $i, j \in H$, the pair of households (i, j) bargain over the risk-sharing contract $(l_{ji}, \gamma_{ij}, \gamma_{ji})$ so as to maximize the bilateral Nash product :

$$\pi_{ij} = \left[u\left(\tilde{c}_i\right) - \bar{u}_{i,-j}\right]\left[u\left(\tilde{c}_j\right) - \bar{u}_{j,-i}\right]. \tag{22}$$

[21] In fact, we will show in this section that households always have an incentive to deviate from the second best outcome.

Gathering the $n(n-1)/2$ bilateral agreements, one obtains the aggregate risk-sharing arrangement $\left(L^{DB},\Gamma^{DB}\right)$ following the same structure as in (2), where, following previous notations, $l_h=\sum_{j\in H\setminus\{h\}} l_{jh}$ and $\alpha_h=\sum_{i\in H\setminus\{h\}}\gamma_{hi}$.

The following proposition describes the solution to the decentralized bargaining problem.

Proposition 6 *Under decentralized bargaining,*

1. *the vector of lump sum transfers L^{DB} is such that for all $i,j\in H$,*

$$\frac{u'\left(\tilde{c}_i\right)}{u'\left(\tilde{c}_j\right)}=\frac{u\left(\tilde{c}_i\right)-\bar{u}_{i,-j}}{u\left(\tilde{c}_j\right)-\bar{u}_{j,-i}},$$

2. *the risk-sharing arrangement Γ^{DB} is such that for all $i,j\in H$,*

$$\gamma_{ij}^{DB} = \frac{\frac{\tau_j}{1+\epsilon_{\sigma_i,\gamma_{ij}}}}{\tau_i+\sum\limits_{h\in H\setminus\{i\}}\frac{\tau_h}{1+\epsilon_{\sigma_i,\gamma_{ih}}}}, \tag{23}$$

$$\alpha_i^{DB} = 1-\frac{1}{1+\frac{1}{\tau_i}\sum\limits_{j\in H\setminus\{i\}}\frac{\tau_j}{1+\epsilon_{\sigma_i,\gamma_{ij}}}}, \tag{24}$$

where $\epsilon_{\sigma_i,\gamma_{ij}}=\frac{\partial\sigma_i^N}{\partial\gamma_{ij}}\frac{\gamma_{ij}}{\sigma_i^N}=\epsilon_i\frac{\gamma_{ij}}{\alpha_i}$,

3. *the risk-taking profile is $\Sigma^{DB}=\Sigma^N\left(\Gamma^{DB}\right)=\left(\sigma_1^{DB},...,\sigma_n^{DB}\right)$, where $\sigma_i^{DB}=\sigma_i^N\left(\alpha_i^{DB}\right)$ is given by*

$$\mu'\left(\sigma_i^{DB}\right)=\tau_i\left(\frac{1}{1+\frac{1}{\tau_i}\sum\limits_{j\in H\setminus\{i\}}\frac{\tau_j}{1+\epsilon_{\sigma_i,\gamma_{ij}}}}\right)^2\sigma_i^{DB}. \tag{25}$$

Proof. Provided in Appendix 4. ■

As can be seen from Proposition 6, the decentralized bargaining (DB) and second best solutions (SB) are relatively similar as they both try to mitigate moral hazard, represented by the presence of ϵ_i in (17) and $\epsilon_{\sigma_i,\gamma_{ij}}=\epsilon_i\frac{\gamma_{ij}}{\alpha_i}$ (24). However, risk externalities are only internalized at the level of the pair $\{i,j\}$ in the decentralized bargaining case. While determining $\left(\gamma_{ij},\gamma_{ji}\right)$, households i and j take into account the impact of their risk-taking on each other's utilities, but neglect its impact on their other partners outside the pair. Since it doesn't sufficiently mitigate moral hazard, γ_{ij}^{DB} is too high, as discussed in the next proposition.

Proposition 7 *Under decentralized bargaining, risk-sharing is lower than the first best level, but higher than the second best level: for all $h\in H$,*

$$\alpha_h^{SB}<\alpha_h^{DB}<\alpha_h^{FB}.$$

Proof. This results obtains by comparing the first order conditions in the various cases. The only difference between them comes from the marginal costs of risk-sharing (externalities) which are highest in the second best and lowest in the first best. ■

As expected, a higher level of risk-sharing results in higher risk-taking as well.

Proposition 8 *Under decentralized bargaining, the risk-taking level is always higher than the second best level ($\sigma_h^{SB}<\sigma_h^{DB}$).*

Proof. This result readily follows from the previous proposition, since σ_h^N is increasing in α_h. ■

Summing up, when a group is characterized by a lack of coordination or by its inability to enforce a centralized cooperative solution, then the moral hazard problem proves more severe, which results in a higher level of risk-sharing and an inefficiently high level of risk-taking.

6 Imposing structure on the composition of groups

In this section, we explore the role of risk attitudes as well as the impact of heterogeneity within risk-sharing groups.

6.1 The case of homogeneous groups

As pointed out in recent empirical works (Ahlin (2010), Giné et al. (2010), Attanasio et al. (2012)), risk-sharing groups tend to be composed of individuals with similar characteristics. We study in this section how groups which are homogeneous in terms of risk aversion behave in terms of risk-sharing and risk-taking. In this context, we describe the mechanisms explaining why poor (risk averse) households tend to share and take less risks than rich households.

Let us consider the polar case in which, inside a given group, all agents have the same degree of risk aversion (risk tolerance): for all $i, j \in H$, $a_i = a_j = a$ ($\tau_i = \tau_j = 1/a$).[22] It is worth noting that independently of the setting (FB, SB and DB), homogeneity implies that $\gamma_{ij} = \gamma\ \forall i \neq j \in H$, and $\alpha_h = \sum_{i \in H \setminus \{h\}} \gamma_{hi} = (n-1)\,\gamma = \alpha$. As a result, the risk-sharing matrix Γ of a homogeneous group is symmetric and has the following structure:

$$\Gamma = \begin{pmatrix} -\alpha & \frac{\alpha}{(n-1)} & \cdots & \frac{\alpha}{(n-1)} \\ \frac{\alpha}{(n-1)} & \ddots & & \vdots \\ \vdots & & \ddots & \frac{\alpha}{(n-1)} \\ \frac{\alpha}{(n-1)} & \cdots & \frac{\alpha}{(n-1)} & -\alpha \end{pmatrix}.$$

One can see from this expression that the insurance scheme is fully characterized by a single parameter, α. Of course, the levels of α depend on the group's risk aversion on the one hand, and on the regime under which the risk-sharing arrangement is established, on the other hand, with

$$\alpha^{FB} = \frac{n-1}{n} > \alpha^{DB} = \frac{n-1}{n + \frac{\epsilon}{n-1}} > \alpha^{SB} = \frac{n-1}{n+\epsilon}.$$

It is already worth noting at this stage that the mechanism through which risk-sharing α is affected by risk aversion a is related to ϵ, the elasticity of individual risk-taking to insurance. As seen in Propositions 3 and 6, the moral hazard problem, which is captured by this elasticity, indeed limits the degree of risk-sharing. In the next proposition, we address the following question: how do groups composed of poor (risk averse) households share their risks compared to rich (less risk averse) groups? In order to answer this question, a first step is therefore to see how risk aversion affects this elasticity ϵ.

Lemma 3 *The elasticity of risk-taking to risk-sharing is increasing in risk aversion:* $\frac{\partial \epsilon}{\partial a} > 0$.

[22] Our previous results in Propositions 1, 3 and 6 can be easily translated to this particular case, so that we won't restate them thoroughly.

Proof. Provided in Appendix 5. ■

This lemma states that the moral hazard problem is more severe among poor (risk averse) households. Indeed, these households' risk-taking behavior is more sensitive to insurance. Note that this result is general and independent of the group composition since it is only based on household optimization for any given insurance arrangement (see equation (2)). This result has important consequences on risk-sharing however, as stated in the next proposition. In order to see this, we analyze the impact of a homogeneous increase in risk aversion (i.e. affecting all group members) on the equilibrium level of risk-sharing.

Proposition 9 ***The impact of risk aversion on risk-sharing in homogeneous groups***
Poor (risk averse) groups, which are more subject to moral hazard, share less risks than rich groups under both second best and decentralized bargaining solutions:

$$\begin{aligned} \frac{d\alpha^{SB}}{da} &= \frac{\partial \alpha^{SB}}{\partial \epsilon}\frac{\partial \epsilon}{\partial a} < 0, \\ \frac{d\alpha^{DB}}{da} &= \frac{\partial \alpha^{DB}}{\partial \epsilon}\frac{\partial \epsilon}{\partial a} < 0. \end{aligned}$$

Proof. Provided in Appendix 5. ■

Lemma 3 and Proposition 9 provide a central result of this paper. We have seen in Lemma 3 that moral hazard is more prevalent in poor groups. Proposition 9 tells us that because of this, the social cost of insurance (risk externalities) is higher in poor groups. As a result, these groups are more reluctant to share risks than rich groups. This result provides a rationale for the fact that poor households are less able to protect themselves against adverse shocks. Let us now analyze whether poor groups also tend to take less risks than rich ones.

Proposition 10 ***The impact of risk aversion on risk-taking in homogeneous groups***
Poor (risk averse) groups take less risks than rich groups under both second best and decentralized bargaining solutions:

$$\begin{aligned} \frac{d\sigma^{SB}}{da} &= \frac{\partial \sigma^{SB}}{\partial a} + \frac{\partial \sigma^{SB}}{\partial \alpha}\frac{\partial \alpha^{SB}}{\partial a} < 0, \\ \frac{d\sigma^{DB}}{da} &= \frac{\partial \sigma^{DB}}{\partial a} + \frac{\partial \sigma^{DB}}{\partial \alpha}\frac{\partial \alpha^{DB}}{\partial a} < 0. \end{aligned}$$

Proof. By applying the implicit function theorem on equation (14), it is straightforward to show that $\frac{\partial \sigma^N}{\partial a} < 0$ and $\frac{\partial \sigma^N}{\partial \alpha} > 0$. Combining with $\frac{\partial \alpha}{\partial a} < 0$ (from Proposition 9), one obtains the result. ■

Two effects differentiate risk-taking between poor and rich groups. First, since the rich are more risk-tolerant, they are ready to take more risks, ceteris paribus. Second, by Proposition 9, we know that rich groups share more risks ($\frac{\partial \alpha^{DB}}{\partial a} < 0$). Because receiving more insurance induces agents to take more risks, this second effect reinforces the first: rich groups take more risks because they have higher risk tolerance and higher risk-sharing.

Taken together, Propositions 9 and 10 therefore rationalize the two stylized facts according to which poor households tend to be more affected by idiosyncratic shocks and are less keen to adopt high risk / high return technologies.

6.2 The case of heterogeneous groups

While the case of homogeneous groups treated above appears realistic, let us analyze for the sake of generality how group heterogeneity affects the relationship between risk aversion and risk-sharing. Let us consider the case of a group composed of n_r rich and n_p poor households, with $a_r < a_p$ and $n_r + n_p = n$. Let R (P) denote the set of rich (poor) households, with $H = R \cup P$. The case of a rich-poor group is a also a particular case of the general model studied above. In this setting, the risk-sharing arrangement can be rewritten as

$$\underset{(n\times n)}{\Gamma} = \begin{pmatrix} \Gamma_{rr} & \Gamma_{rp} \\ \Gamma_{pr} & \Gamma_{pp} \end{pmatrix},$$

where

$$\underset{(n_r\times n_r)}{\Gamma_{rr}} = \begin{pmatrix} -\alpha_r & \gamma_{rr} & \cdots & \gamma_{rr} \\ \gamma_{rr} & \ddots & & \vdots \\ \vdots & & \ddots & \gamma_{rr} \\ \gamma_{rr} & \cdots & \gamma_{rr} & -\alpha_r \end{pmatrix}; \quad \underset{(n_p\times n_p)}{\Gamma_{pp}} = \begin{pmatrix} -\alpha_p & \gamma_{pp} & \cdots & \gamma_{pp} \\ \gamma_{pp} & \ddots & & \vdots \\ \vdots & & \ddots & \gamma_{pp} \\ \gamma_{pp} & \cdots & \gamma_{pp} & -\alpha_p \end{pmatrix}.$$

The structure of Γ_{rr} (Γ_{pp}) is very similar to the structure of a risk-sharing arrangement in a homogeneous rich (poor) group. In this sense, these submatrices can be interpreted as the risk-sharing arrangements within each subgroup (R and P). However, contrary to the analysis of the homogeneous group, the rich also share risks with the poor, so that the budget constraint within a subgroup need not be satisfied. Instead, since the constraint imposes budget balance at the level of the entire group H, one subgroup may enjoy a surplus which is financed by the other subgroup. The cross-group risk-sharing arrangements are represented by the submatrices Γ_{rp} and Γ_{pr}, which write

$$\underset{(n_r\times n_p)}{\Gamma_{rp}} = \begin{pmatrix} \gamma_{rp} & \cdots & \gamma_{rp} \\ \vdots & \ddots & \vdots \\ \gamma_{rp} & \cdots & \gamma_{rp} \end{pmatrix}; \quad \underset{(n_p\times n_r)}{\Gamma_{pr}} = \begin{pmatrix} \gamma_{pr} & \cdots & \gamma_{pr} \\ \vdots & \ddots & \vdots \\ \gamma_{pr} & \cdots & \gamma_{pr} \end{pmatrix}.$$

Since there are only two types of agents, risk-sharing across groups can be summarized by two parameters, γ_{rp} and γ_{pr}. Applying this structure to Lemma 1, the budget constraint imposes that

$$\alpha_r = (n_r - 1)\gamma_{rr} + n_p\gamma_{rp}, \tag{26}$$

$$\alpha_p = (n_p - 1)\gamma_{pp} + n_r\gamma_{pr}. \tag{27}$$

When a poor household faces a shock, it is insured against a fraction α_p of this shock. This fraction is absorbed by the $(n_p - 1)$ other poor households at a rate of γ_{pp}, and by the n_r rich households at a rate of γ_{pr}. The parameter $\gamma_{ww'}$ can therefore be interpreted as the fraction of a w household's shock which is absorbed by a w' household.

Let us now analyze the impact of group heterogeneity on the relationship between risk aversion and risk-sharing. To do so, we study the comparative statics of the coefficient of absolute risk-aversion of the poor a_p. More precisely, we analyze how the risk-sharing arrangement $(\alpha_r, \gamma_{rr}, \gamma_{rp}, \alpha_p, \gamma_{pp}, \gamma_{pr})$ evolves as the n_p poor households become more risk averse. Since initially $a_p \geq a_r$, such a variation represents an increase in the degree of heterogeneity of the group.

Proposition 11 ***The impact of group heterogeneity on the decentralized risk-sharing arrangement***

An increase in poor households' risk aversion has a negative impact on risk-sharing for the rich, and an *ambiguous impact for the poor:*

$$\frac{d\alpha_r^{DB}}{da_p} < 0,$$
$$\frac{d\alpha_p^{DB}}{da_p} \gtrless 0.$$

Proof. Provided in Appendix 6. ■

Proposition 11 provides contrasting predictions about the impact of group heterogeneity on the insurance coverage that informal risk-sharing offers to the rich and to the poor. Indeed, while an increase in the poor subgroup's risk aversion leads to a reduction in risk-sharing for rich households, its effect is indeterminate for the poor. Let us describe the mechanisms behind this twofold result, starting with the impact on α_r.

Using the budget constraint and equation (26), one can see that a variation in a_p affects α_r through γ_{rr} and γ_{rp}:

$$\frac{d\alpha_r}{da_p} = (n_r - 1)\frac{d\gamma_{rr}}{da_p} + n_p\frac{d\gamma_{rp}}{da_p}.$$

Since a variation in a_p affects both γ_{rr} and γ_{rp} simultaneously, total effects $\frac{d\gamma_{rr}}{da_p}$ and $\frac{d\gamma_{rp}}{da_p}$ are rather complex. To illustrate this, let us start by decomposing in detail the total effect of a_p on γ_{rr}. First, there is a direct (partial) effect of a_p on γ_{rr}, which in this particular case is zero because the poor's risk aversion does not directly affect bargaining between two rich households. Second, there is an indirect effect through γ_{rp}: when a_p increases, γ_{rp} (the insurance offered by the poor to the rich) decreases. This decrease in γ_{rp} in turn affects the way two rich negotiate, but its impact on γ_{rr} is indeterminate.

The case of $\frac{d\gamma_{rp}}{da_p}$ presents the converse situation: the direct effect of a_p on γ_{rp} is negative, while the indirect effect through γ_{rr} is zero. As a result, $\frac{d\gamma_{rp}}{da_p}$ is always negative.

Summing up for the total effect of a_p on α_r, while the first effect on the "within rich subgroup" risk-sharing, $\frac{d\gamma_{rr}}{da_p}$, is ambiguous, the second effect $\frac{d\gamma_{rp}}{da_p}$ is negative. Interestingly, the direct effect on γ_{rp} always dominates the indirect effect on γ_{rr} (through γ_{rp}) so that the total effect on α_r is always negative.

Let us now analyze the effect of a_p on the poor's risk coverage α_p. Using equation (27), one can decompose this effect into:

$$\frac{d\alpha_p}{da_p} = (n_p - 1)\frac{d\gamma_{pp}}{da_p} + n_r\frac{d\gamma_{pr}}{da_p}.$$

This part of the analysis does not provide clear results since both total effects, $d\gamma_{pp}^{DB}/da_p$ and $d\gamma_{pr}^{DB}/da_p$ are indeterminate. Yet, it is interesting to note that, when these two indeterminate effects are combined to compute $\frac{d\alpha_p}{da_p}$, the indirect effects on γ_{pp} (through γ_{pr}) and on γ_{pr} (through γ_{pp}) cancel out, which simplifies the interpretation of the results. Indeed, $d\alpha_p/da_p$ depends only on the direct effects of a_p on γ_{pp} and γ_{pr}, which we describe here.

First, an increase in a_p unambiguously decreases the level of "within poor subgroup" risk-sharing ($\partial\gamma_{pp}^{DB}/\partial a_p > 0$). The mechanism behind this result is that an increase in risk aversion affects ϵ_p. Following Lemma 3, we indeed know that an increase in risk aversion strengthens the moral hazard problem, which discourages poor households from sharing risks with each other, in line with the result obtained in homogeneous groups.

As regards $\partial\gamma_{pr}^{DB}/\partial a_p$, its sign is indeterminate since it results from two conflicting effects. On the one hand, if they become more risk averse, the poor's marginal benefit of being insured by the rich increases. On the other hand, moral hazard by the poor is stronger, which deters the rich from offering them insurance.

Since the moral hazard problem affects the willingness of both poor and rich to offer insurance to the poor, a total negative effect of a_p on α_p remains likely, as in the case of homogeneous group.

Finally, let us interpret Proposition 11 in terms of inter-group comparison. Consider for instance a rich household that would be initially a member of a homogeneous (rich) group. Proposition 11 tells us that, in a group of identical size but composed of a certain number of slightly more risk averse households, this rich household would have received a lower level of insurance. This result therefore indicates that rich households may be reluctant to share risk with poorer households, which may provide an additional rationale to the fact that groups tend to be formed homogeneously in terms of wealth or risk attitudes.

7 Concluding remarks

The analysis conducted in this paper has been motivated by a twofold empirical observation: on the one hand, poor farm households are reluctant to adopt technologies and to take risks that would allow them to obtain higher returns. On the other hand, despite taking less risks, they tend to suffer from a higher exposure to idiosyncratic shocks. We show that some imperfections which are inherent to informal risk-sharing arrangements, namely small group sizes and missing insurance and capital markets, are particularly detrimental to the poor and may contribute to explaining those stylized facts.

Within this set of imperfections, we have analyzed the interactions between households' risk-taking behavior and risk-sharing arrangements under various regimes, i.e. second best and decentralized bargaining. Our representation of risk-taking strategies is based on a trade-off between return (average of income) and risk (variance of income). While a standard setting with formal markets leads to efficient outcomes under this representation, this is not the case under the specific framework of informal risk-sharing. On the one hand, due to their reduced size, informal groups achieve a limited level of risk diversification. On the other hand, missing capital and reinsurance markets imply that the group's transfers must adapt to all potential realizations of shocks. As a result, individual behavior affects all group members through the mutual insurance scheme, leading to risk externalities. These externalities, which are present independently of the regime under which the insurance arrangement is set, naturally lead to moral hazard problems in terms of excessive risk-taking.

We first show that any cooperative insurance arrangement (second best or decentralized bargaining) limits the degree of risk-sharing in order to mitigate moral hazard.

Second, in addition to the classical first-best / second-best analysis, we also formalize the risk-sharing process as a decentralized bargaining in which all households are free to negotiate the terms of risk-sharing arrangements with any potential household. This solution concept appears more realistic in an environment where enforcement devices are lacking, as it does not rely on the strong coordination imposed by the classical second best approach. The comparison between this setting and the second best provides interesting insights. When a pair of households bargains over insurance transfers, they anticipate that these transfers will affect each other's risk-taking behavior. However, they do not internalize the externalities that this risk-taking generates on their other partners with whom they also share risks. As a result, the lack of coordination at the group level strengthens the moral hazard mechanism.

Third, we investigate the role played by risk aversion and the implications of group heterogeneity on risk-taking and risk-sharing. Interestingly, we show that the moral hazard problem is stronger for poor households, whose risk-taking behavior is more sensitive to insurance. This leads to the result that, if insurance groups are homogeneous in terms of wealth or risk aversion, poor groups share less risks than rich groups.

Analyzing the impact of group heterogeneity, we show that the rich's insurance tends to decrease in the presence of poor households. This suggests that the rich might be reluctant to share risk with poor households. Furthermore, it may also be that an increase in the group's risk aversion also penalizes the poor. Indeed, despite the gains of sharing risks with the rich, their insurance level might decreases due to the moral hazard problem. While these results shed light on the benefits of group homogeneity in informal risk-sharing, a deeper exploration of the process of group formation in presence of moral hazard appears an interesting research avenue.

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8 Appendix 1: proof of Lemma 1

For the budget constraint to be satisfied with equality, the sum of transfers over the whole group, which we denote by $B(S)$, needs to be zero for all $S \in R^n$. Making use of equation (2), the budget constraint can be written as

$$B(S) = \sum_{h \in H} t_h(S) = 0 \iff \sum_{h \in H} \left[l_h + \sum_{j \in H} \gamma_{jh} s_j \right] = 0, \forall S \in R^n$$

First notice that in the particular case where $S = (0, ..., 0)'$, this condition implies that $\sum_{h \in H} l_h = 0$. Therefore, we can rewrite the budget constraint as

$$\begin{aligned} B(S) \quad &= \quad 0 \iff \sum_{h \in H} \sum_{j \in H} \gamma_{jh} s_j = 0, \forall S \in R^n \\ &\iff \quad s_1 \sum_{h \in H} \gamma_{1h} + ... + s_n \sum_{h \in H} \gamma_{nh} = 0, \forall S \in R^n. \end{aligned} \tag{28}$$

Therefore, in the particular case where $S = (s_1, 0, ..., 0)'$, with $s_1 \neq 0$, condition (28) is satisfied if and only if $\sum_{h \in H} \gamma_{1h} = 0$. Similarly, when $S = (0, s_2, 0, ..., 0)'$, with $s_2 \neq 0$, we need to have $\sum_{h \in H} \gamma_{2h} = 0$, etc. Applying the same reasoning to the n income shocks gives the condition of Lemma 1.

9 Appendix 2: proof of Proposition 1

The first best can be found by maximizing the social welfare function (9) with respect to the parameters of the transfer scheme (L, Γ) and the risk-taking profile Σ.

The first order condition with respect to any lumpsum transfer l_{ij} imposes that

$$\frac{\partial W}{\partial l_{ij}} = 0 \iff \lambda_i u'(\tilde{c}_i) = \lambda_j u'(\tilde{c}_j),$$

which gives the first point.

Making use of equation (10) and of the expression of the consumption variance (6), the first order condition with respect to any γ_{ij} can be written as

$$\begin{aligned} \frac{\partial W}{\partial \gamma_{ij}} &= 0 \iff \frac{\partial \tilde{c}_i}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_j}{\partial \gamma_{ij}} = 0 \\ &\iff a_i (1 - \alpha_i) = a_j \gamma_{ij}^{FB}. \end{aligned} \tag{29}$$

Recall that, by definition, $\alpha_i = \sum_{j \in H \setminus \{i\}} \gamma_{ij}$. Summing over all $j \in H \setminus \{i\}$,

$$\alpha_i^{FB} = \sum_{j \in N \setminus \{i\}} \frac{a_i}{a_j} \left(1 - \alpha_i^{FB}\right).$$

Solving for α_i, we end up with

$$\alpha_i^{FB} = 1 - \frac{\tau_i}{\sum_{h \in H} \tau_h},$$

with $\tau = 1/a$.

Using this expression in equation (29), allows to find the first best value of any given γ_{ij}:

$$\gamma_{ij}^{FB} = \frac{a_i}{a_j} \left(1 - \alpha_i^{FB}\right) = \frac{\tau_j}{\sum_{h \in H} \tau_h}.$$

Finally, the first order condition with respect to risk-taking σ_i imposes that

$$\frac{\partial W}{\partial \sigma_i} = 0 \iff \mu'\left(\sigma_i^{FB}\right) - a_i (1 - \alpha_i)^2 \sigma_i^{FB} - \sum_{j \in H \setminus \{i\}} a_j \gamma_{ij}^2 \sigma_i^{FB} = 0.$$

Rearranging and substituting for the first best values of Γ gives the result.

10 Appendix 3: proof of Propositions 3 and 5

In order to find the second best, we maximize the social welfare function (9) with respect to the transfer scheme (L, Γ), subject to the incentive compatibility condition (14), which gives us a function $\sigma_h^N(\alpha_h), \forall h \in H$.

First, the first order condition with respect to the lump sum transfer is unchanged as compared to the first best analysis (equation 10).

Second, making use of the latter condition, the first order condition with respect to any γ_{ij} can be written as

$$\begin{aligned} \frac{\partial W}{\partial \gamma_{ij}} &= 0 \iff \sum_{h \in H} \frac{d\tilde{c}_h}{d\gamma_{ij}} = 0 \\ &\iff \left(\frac{\partial \tilde{c}_i}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_i}{\partial \sigma_i}\frac{\partial \sigma_i^N}{\partial \gamma_{ij}}\right) + \left(\frac{\partial \tilde{c}_j}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_j}{\partial \sigma_i}\frac{\partial \sigma_i^N}{\partial \gamma_{ij}}\right) + \sum_{h \in H \setminus \{i,j\}} \left(\frac{\partial \tilde{c}_h}{\partial \sigma_i}\frac{\partial \sigma_i^N}{\partial \gamma_{ij}}\right) = 0, \end{aligned}$$

where $\partial \tilde{c}_i / \partial \sigma_i = 0$, by the envelope theorem, and where $\partial \sigma_i^N / \partial \gamma_{ij} = \partial \sigma_i^N / \partial \alpha_i$, since $\alpha_i = \sum_{j \in H \backslash \{i\}} \gamma_{ij}$. Therefore,

$$\frac{\partial W}{\partial \gamma_{ij}} = 0 \iff \frac{\partial \tilde{c}_i}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_j}{\partial \gamma_{ij}} + \sum_{h \in H \backslash \{i\}} \left(\frac{\partial \tilde{c}_h}{\partial \sigma_i} \frac{\partial \sigma_i^N}{\partial \gamma_{ij}} \right) = 0.$$

Making use of the expression of the consumption variance (6), we end up with

$$\begin{aligned} \frac{\partial W}{\partial \gamma_{ij}} &= 0 \iff a_i \left(1 - \alpha_i\right) \sigma_i^2 - a_j \gamma_{ij} \sigma_i^2 - \sum_{h \in H \backslash \{i\}} a_h \gamma_{ih}^2 \sigma_i \frac{\partial \sigma_i^N}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial \gamma_{ij}} = 0 \\ &\iff a_i \left(1 - \alpha_i\right) - a_j \gamma_{ij} - \sum_{h \in H \backslash \{i\}} a_h \frac{\gamma_{ih}^2}{\alpha_i} \epsilon_i = 0. \end{aligned} \tag{30}$$

Summing over all $j \in H \backslash \{i\}$, we find

$$\begin{aligned} (n-1)\, a_i \left(1 - \alpha_i\right) &= \sum_{j \in H \backslash \{i\}} a_j \gamma_{ij} + (n-1) \sum_{h \in H \backslash \{i\}} a_h \frac{\gamma_{ih}^2}{\alpha_i} \epsilon_i \\ 1 - \alpha_i &= \sum_{h \in H \backslash \{i\}} \gamma_{ih} \frac{\tau_i}{\tau_h} \left(\frac{1}{n-1} + \frac{\gamma_{ih}}{\alpha_i} \epsilon_i \right), \end{aligned}$$

where $\tau_h = 1/a_h$. Using $\alpha_i = \sum_{h \in N \backslash \{i\}} \gamma_{ih}$, we obtain

$$\sum_{h \in H \backslash \{i\}} \gamma_{ih} \left(1 + \frac{\tau_i}{\tau_h} \frac{1}{n-1} + \frac{\tau_i}{\tau_h} \frac{\gamma_{ih}}{\alpha_i} \epsilon_i \right) = 1. \tag{31}$$

This leads to

$$\gamma_{ij}^{SB} = \frac{\tau_j}{\sum_{h \in H} \tau_h + \tau_i \epsilon_i}.$$

With this expression of γ_{ij}, the condition in (31) is indeed satisfied. To see this, let us compute

$$\begin{aligned} & \sum_{j \in H \backslash \{i\}} \frac{\tau_j}{\sum_{h \in H} \tau_h + \tau_i \epsilon_i} \left(1 + \frac{\tau_i}{\tau_j} \frac{1}{n-1} + \frac{\tau_i}{\tau_j} \frac{\gamma_{ij}}{\alpha_i} \epsilon_i \right) \\ = \;& \frac{1}{\sum_{h \in H} \tau_h + \tau_i \epsilon_i} \left(\sum_{j \in H \backslash \{i\}} \tau_j + \tau_i + \sum_{j \in H \backslash \{i\}} \tau_i \frac{\gamma_{ij}}{\alpha_i} \epsilon_i \right) = 1, \end{aligned}$$

by definition of α_i. Note that the initial first order condition in equation (19) is also valid under this solution:

$$\begin{aligned}
a_i\left(1-\alpha_i\right) &= a_j\gamma_{ij}+\sum_{h\in H\backslash\{i\}} a_h\frac{\gamma_{ih}^2}{\alpha_i}\epsilon_i=0\\
a_i\left(\frac{\tau_i\left(1+\epsilon_i\right)}{\sum\limits_{h\in H}\tau_h+\tau_i\epsilon_i}\right) &= a_j\frac{\tau_j}{\sum\limits_{h\in H}\tau_h+\tau_i\epsilon_i}+\sum_{h\in H\backslash\{i\}} a_h\left(\frac{\tau_h}{\sum\limits_{h\in H}\tau_h+\tau_i\epsilon_i}\right)^2\frac{1}{\sum\limits_{h\in H\backslash\{i\}}\left(\frac{\tau_h}{\sum_{h\in H}\tau_h+\tau_i\epsilon_i}\right)}\epsilon_i\\
\left(\frac{\left(1+\epsilon_i\right)}{\sum\limits_{h\in H}\tau_h+\tau_i\epsilon_i}\right) &= \frac{1}{\sum\limits_{h\in H}\tau_h+\tau_i\epsilon_i}+\epsilon_i\left(\frac{1}{\sum\limits_{h\in H}\tau_h+\tau_i\epsilon_i}\right)\frac{\sum\limits_{h\in H\backslash\{i\}}\tau_h}{\sum\limits_{h\in H\backslash\{i\}}\tau_h}\\
1+\epsilon_i &= 1+\epsilon_i
\end{aligned}$$

The expression of α_i^{SB} can then be found by using the definition of α_i and the second best value of γ_{ij}.

Third, the condition for the second best level of risk-taking for household $i\in H$ comes from the expression of the Nash equilibrium (14), where α is replaced by its second best value.

Finally, we compare second best to first best risk-taking (Proposition 5).

To prove the condition on risk-taking, start by noting that $\frac{\mu'(\sigma)}{\sigma}$ is a decreasing function of σ. As a result, $\sigma_h^{SB}<\sigma_h^{FB}\iff\mu'\left(\sigma_h^{SB}\right)/\sigma_h^{SB}>\mu'\left(\sigma_h^{FB}\right)/\sigma_h^{FB}$. Substituting from Propositions 1 and 3 (equations 12 and 18),

$$\begin{aligned}
\sigma_h^{SB} &< \sigma_h^{FB}\iff\tau_h\left(\frac{1+\epsilon_h}{\sum\limits_{i\in H}\tau_i+\tau_h\epsilon_h}\right)^2>\frac{1}{\sum\limits_{i\in H}\tau_i}\\
&\iff \sum_{i\in H}\frac{\tau_i}{\tau_h}\left(1+\epsilon_h\right)^2>\left(\sum_{i\in H}\frac{\tau_i}{\tau_h}+\epsilon_h\right)^2.
\end{aligned}$$

Rearranging,

$$\begin{aligned}
\sigma_h^{SB} &< \sigma_h^{FB}\iff\left(\epsilon_i^2-\sum_{i\in H}\frac{\tau_i}{\tau_h}\right)\left(\sum_{i\in H}\frac{\tau_i}{\tau_h}-1\right)>0\\
&\iff \epsilon_i^2>\sum_{i\in H}\frac{\tau_i}{\tau_h}.
\end{aligned}$$

11 Appendix 4: proof of Proposition 6

The solution to the Nash bargaining problem can be found by maximizing the Nash product (22) with respect to the terms of the bilateral risk-sharing arrangement $\left\{l_{ji},\gamma_{ij},\gamma_{ji}\right\}$.

The first order condition with respect to the lump sum transfer l_{ij} imposes that

$$\frac{\partial\pi_{ij}}{\partial l_{ij}}=0\iff u'\left(\tilde{c}_i\right)\left[u\left(\tilde{c}_i\right)-\bar{u}_{i,-j}\right]^{-1}=\left[u\left(\tilde{c}_j\right)-\bar{u}_{j,-i}\right]^{-1}u'\left(\tilde{c}_j\right).$$

Making use of the latter condition, the first order condition with respect to γ_{ij} (we proceed similarly for γ_{ji}) can be directly written as

$$\begin{aligned}\frac{\partial \pi_{ij}}{\partial \gamma_{ij}} &= 0 \iff \frac{d\tilde{c}_i}{d\gamma_{ij}} + \frac{d\tilde{c}_j}{d\gamma_{ij}} = 0 \\ &\iff \left(\frac{\partial \tilde{c}_i}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_i}{\partial \sigma_i}\frac{\partial \sigma_i^N}{\partial \gamma_{ij}}\right) + \left(\frac{\partial \tilde{c}_j}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_j}{\partial \sigma_i}\frac{\partial \sigma_i^N}{\partial \gamma_{ij}}\right) = 0,\end{aligned}$$

where $\partial \tilde{c}_i/\partial \sigma_i = 0$, by the envelope theorem, and where $\partial \sigma_i^N/\partial \gamma_{ij} = \partial \sigma_i^N/\partial \alpha_i$, since $\alpha_i = \sum_{j \in H\backslash\{i\}} \gamma_{ij}$. Therefore,

$$\frac{\partial \pi_{ij}}{\partial \gamma_{ij}} = 0 \iff \frac{\partial \tilde{c}_i}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_j}{\partial \gamma_{ij}} + \frac{\partial \tilde{c}_j}{\partial \sigma_i}\frac{\partial \sigma_i^N}{\partial \gamma_{ij}} = 0.$$

Making use of the expression of the consumption variance (6), we end up with

$$\begin{aligned}\frac{\partial \pi_{ij}}{\partial \gamma_{ij}} &= 0 \iff a_i\left(1-\alpha_i\right)\sigma_i^2 - a_j\gamma_{ij}\sigma_i^2 - a_j\gamma_{ij}^2\sigma_i\frac{\partial \sigma_i^N}{\partial \alpha_i} = 0 \\ &\iff a_i\left(1-\alpha_i\right) - a_j\gamma_{ij}\left(1+\frac{\gamma_{ij}}{\alpha_i}\epsilon_i\right) = 0 \\ &\iff a_i\left(1-\alpha_i\right) - a_j\gamma_{ij}\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right) = 0,\end{aligned} \tag{32}$$

where $\epsilon_{\sigma_i,\gamma_{ij}} = \frac{\gamma_{ij}}{\sigma_i}\frac{\partial \sigma_i^N}{\partial \gamma_{ij}} = \epsilon_{\sigma_i,\alpha_i}\frac{\gamma_{ij}}{\alpha_i}$. Rearranging, one obtains

$$\gamma_{ij} = \frac{a_i\left(1-\alpha_i\right)}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)}. \tag{33}$$

Summing over all $j \in H\backslash\{i\}$, we find

$$\begin{aligned}\sum_{j\in H\backslash\{i\}} \gamma_{ij} &= \sum_{j\in H\backslash\{i\}} \frac{a_i\left(1-\alpha_i\right)}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)} \iff \\ \alpha_i &= a_i\left(1-\alpha_i\right)\sum_{j\in H\backslash\{i\}} \frac{1}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)} \iff \\ \alpha_i\left[1 + a_i\sum_{j\in H\backslash\{i\}} \frac{1}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)}\right] &= a_i\sum_{j\in H\backslash\{i\}} \frac{1}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)} \iff \\ \alpha_i^{DB} &= \frac{a_i\sum_{j\in H\backslash\{i\}} \frac{1}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)}}{1 + a_i\sum_{j\in H\backslash\{i\}} \frac{1}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)}}.\end{aligned}$$

Rearranging, one obtains

$$\alpha_i^{DB} = 1 - \left(\frac{\tau_i}{\tau_i + \sum_{j\in H\backslash\{i\}} \frac{\tau_j}{1+\epsilon_{\sigma_i,\gamma_{ij}}}}\right).$$

It follows that, using (33),

$$\gamma_{ij}^{DB} = \frac{a_i}{a_j\left(1+\epsilon_{\sigma_i,\gamma_{ij}}\right)}\left(\frac{\tau_i}{\tau_i + \sum_{j\in H\backslash\{i\}} \frac{\tau_j}{1+\epsilon_{\sigma_i,\gamma_{ij}}}}\right) = \frac{\frac{\tau_j}{1+\epsilon_{\sigma_i,\gamma_{ij}}}}{\tau_i + \sum_{j\in H\backslash\{i\}} \frac{\tau_j}{1+\epsilon_{\sigma_i,\gamma_{ij}}}}.$$

12 Appendix 5: proof of Lemma 3 and Proposition 9

Let us first prove Lemma 3. Making use of equation (15), we have that

$$\frac{\partial \epsilon}{\partial \tau} = \frac{\mu''\left(\sigma_h^N\right) 2\left(1-\alpha_h\right)\alpha_h}{\left(-\mu''\left(\sigma_h^N\right)\tau_h + \left(1-\alpha_h\right)^2\right)^2} < 0.$$

Note that $\frac{\partial \epsilon}{\partial a} = \frac{\partial \epsilon}{\partial \tau}\frac{\partial \tau}{\partial a} = -\frac{\partial \epsilon}{\partial \tau}\frac{1}{a^2} > 0$.

Let us now prove Proposition 9. One can rewrite equations (17) and (24) for homogeneous groups as

$$\begin{aligned} \alpha_i^{SB}\left(n+\epsilon\left(\alpha_i^{SB},a\right)\right) - (n-1) &= 0, \\ \alpha_i^{DB}\left(n+\frac{\epsilon\left(\alpha_i^{DB},a\right)}{n-1}\right) - (n-1) &= 0. \end{aligned}$$

where ϵ is a function of α (and a), as can be seen from equation (15), while a only affects α through ϵ. Therefore, we need to apply the implicit function theorem in order to assess the impact of ϵ on α. This gives

$$\begin{aligned} \frac{\partial \alpha^{SB}}{\partial a} &= -\frac{\alpha_i^{SB}\left(n+\frac{\partial \epsilon}{\partial a}\right) - (n-1)}{(n+\epsilon) + \alpha_i^{SB}\left(n+\epsilon\left(\alpha_i^{SB},a\right)\right)} < 0 \iff \frac{\partial \epsilon}{\partial a} > 0 \iff \frac{\partial \epsilon}{\partial \tau} < 0, \\ \frac{\partial \alpha^{DB}}{\partial a} &< 0 \iff \frac{\partial \epsilon}{\partial a} > 0 \iff \frac{\partial \epsilon}{\partial \tau} < 0. \end{aligned}$$

13 Appendix 6: proof of Proposition 11

In the two-type case, the risk-sharing arrangement Γ is determined by two pairs of parameters: $\left\{\left(\gamma_{rr},\gamma_{rp}\right),\left(\gamma_{pp},\gamma_{pr}\right)\right\}$. Indeed, the other two key parameters, α_r and α_p are characterized separately by these two pairs through the budget constraint: $\alpha_r = (n_r-1)\gamma_{rr} + n_p\gamma_{rp}$, and $\alpha_p = (n_p-1)\gamma_{pp} + n_r\gamma_{pr}$ (equations (26) and (27)). Furthermore, these two pairs of parameters do not depend on each other, since for instance, neither γ_{pp} nor γ_{pr} appear in the first order conditions on γ_{rr} and on γ_{rp}. Indeed, following equation (32), one can write:

$$\begin{aligned} \frac{\partial \pi_{rr}}{\partial \gamma_{rr}} &= 0 \iff \Omega_{rr}\left(\gamma_{rr},\alpha_r\left(\gamma_{rr},\gamma_{rp}\right);a_r\right) = 0, \\ \frac{\partial \pi_{rp}}{\partial \gamma_{rp}} &= 0 \iff \Omega_{rp}\left(\gamma_{rp},\alpha_r\left(\gamma_{rr},\gamma_{rp}\right);a_r,a_p\right) = 0, \end{aligned}$$

where

$$\Omega_{rr}\left(\gamma_{rr},\alpha_r\left(\gamma_{rr},\gamma_{rp}\right);a_r\right) \equiv (1-\alpha_r)\alpha_r - \gamma_{rr}\left(\alpha_r + \gamma_{rr}\epsilon_r\left(\alpha_r;a_r\right)\right) = 0, \tag{34}$$

$$\Omega_{rp}\left(\gamma_{rp},\alpha_r\left(\gamma_{rr},\gamma_{rp}\right);a_r,a_p\right) \equiv a_r(1-\alpha_r)\alpha_r - a_p\gamma_{rp}\left(\alpha_r + \gamma_{rp}\epsilon_r\left(\alpha_r;a_r\right)\right) = 0. \tag{35}$$

The system of two equations (34)-(35) therefore characterizes $\left\{\gamma_{rr}^{DB},\gamma_{rp}^{DB}\right\}$ (and as a result α_r through the budget constraint). Similarly, $\left\{\gamma_{pp}^{DB},\gamma_{pr}^{DB}\right\}$ is characterized by

$$\frac{\partial \pi_{pp}}{\partial \gamma_{pp}} = 0 \iff \Omega_{pp}\left(\gamma_{pp},\alpha_p\left(\gamma_{pp},\gamma_{pr}\right);a_p\right) = (1-\alpha_p)\alpha_p - \gamma_{pp}\left(\alpha_p + \gamma_{pp}\epsilon_p\right) = 0, \tag{36}$$

$$\frac{\partial \pi_{rp}}{\partial \gamma_{pr}} = 0 \iff \Omega_{pr}\left(\gamma_{pr},\alpha_p\left(\gamma_{pp},\gamma_{pr}\right);a_p,a_r\right) = a_p(1-\alpha_p)\alpha_p - a_r\gamma_{pr}\left(\alpha_p + \gamma_{pr}\epsilon_p\right) = 0. \tag{37}$$

In other words, the equilibrium is characterized by two independent systems of two equations, in the sense that the endogenous variables of a system do not interact with the other system.[23] As a consequence, comparative statics of the equilibrium can be achieved by applying the 2 equation, 2 unknown version of the implicit function theorem (IFT) to each of the two separate pairs of first order conditions. For instance, applying the IFT on equations (34)-(35) allows us to determine the impact of a_p on $\left\{\gamma_{rr}^{DB}, \gamma_{rp}^{DB}\right\}$:

$$\frac{d\gamma_{rr}^{DB}}{da_p} = -\frac{\left|H^r_{\left(a_p,\gamma_{rp}\right)}\right|}{\left|H^r_{\left(\gamma_{rr},\gamma_{rp}\right)}\right|}, \tag{38}$$

$$\frac{d\gamma_{rp}^{DB}}{da_p} = -\frac{\left|H^r_{\left(a_p,\gamma_{rr}\right)}\right|}{\left|H^r_{\left(\gamma_{rp},\gamma_{rr}\right)}\right|}, \tag{39}$$

where the matrices $H^r_{\left(a_p,\gamma_{rp}\right)}$, $H^r_{\left(\gamma_{rr},a_p\right)}$ and $H^r_{\left(\gamma_{rr},\gamma_{rp}\right)}$ are composed of derivatives of the system (34)-(35):

$$\begin{aligned}
H^r_{\left(a_p,\gamma_{rp}\right)} &\equiv \begin{pmatrix} \frac{d\Omega_{rr}}{da_p} & \frac{d\Omega_{rp}}{da_p} \\ \frac{d\Omega_{rr}}{d\gamma_{rp}} & \frac{d\Omega_{rp}}{d\gamma_{rp}} \end{pmatrix}, \quad H^r_{\left(a_p,\gamma_{rr}\right)} \equiv \begin{pmatrix} \frac{d\Omega_{rr}}{da_p} & \frac{d\Omega_{rp}}{da_p} \\ \frac{d\Omega_{rr}}{d\gamma_{rr}} & \frac{d\Omega_{rp}}{d\gamma_{rr}} \end{pmatrix}, \\
H^r_{\left(\gamma_{rr},\gamma_{rp}\right)} &\equiv \begin{pmatrix} \frac{d\Omega_{rr}}{d\gamma_{rr}} & \frac{d\Omega_{rp}}{d\gamma_{rr}} \\ \frac{d\Omega_{rr}}{d\gamma_{rp}} & \frac{d\Omega_{rp}}{d\gamma_{rp}} \end{pmatrix}, \quad H^r_{\left(\gamma_{rp},\gamma_{rr}\right)} \equiv \begin{pmatrix} \frac{d\Omega_{rr}}{d\gamma_{rp}} & \frac{d\Omega_{rp}}{d\gamma_{rp}} \\ \frac{d\Omega_{rr}}{d\gamma_{rr}} & \frac{d\Omega_{rp}}{d\gamma_{rr}} \end{pmatrix}.
\end{aligned}$$

Similarly, the impact of a_p on $\left\{\gamma_{pp}^{DB}, \gamma_{pr}^{DB}\right\}$ is determined by the application of the implicit function theorem to the system (36)-(37):

$$\frac{d\gamma_{pp}^{DB}}{da_p} = -\frac{\left|H^p_{\left(a_p,\gamma_{pr}\right)}\right|}{\left|H^p_{\left(\gamma_{pp},\gamma_{pr}\right)}\right|}, \tag{40}$$

$$\frac{d\gamma_{pr}^{DB}}{da_p} = -\frac{\left|H^p_{\left(a_p,\gamma_{pp}\right)}\right|}{\left|H^p_{\left(\gamma_{pr},\gamma_{pp}\right)}\right|}, \tag{41}$$

where $H^p_{\left(a_p,\gamma_{pr}\right)}$, $H^p_{\left(\gamma_{pp},a_p\right)}$, $H^p_{\left(\gamma_{pr},\gamma_{pp}\right)}$ and $H^p_{\left(\gamma_{pp},\gamma_{pr}\right)}$, which are composed of derivatives of the system (36)-(37), are defined in a similar way. Note that since $\left\{\left(\gamma_{rr}^{DB}, \gamma_{rp}^{DB}\right), \left(\gamma_{pp}^{DB}, \gamma_{pr}^{DB}\right)\right\}$ maximize the Nash product π, the determinants $\left|H^r_{\left(\gamma_{rr},\gamma_{rp}\right)}\right| = -\left|H^r_{\left(\gamma_{rp},\gamma_{rr}\right)}\right|$ and $\left|H^p_{\left(\gamma_{pp},\gamma_{pr}\right)}\right| = -\left|H^p_{\left(\gamma_{pr},\gamma_{pp}\right)}\right|$ must be positive. We are now ready to prove the results.

[23] Note however that the exogenous parameter a_p appears in both systems through Ω_{pp} and Ω_{pr} for the poor, and through Ω_{rp} for the rich.

13.1 Comparative statics of α_r^{DB}

Making use of the budget constraint (26) and (38)-(39), and using $\left|H^r_{\left(\gamma_{rr},\gamma_{rp}\right)}\right| = -\left|H^r_{\left(\gamma_{rp},\gamma_{rr}\right)}\right| > 0$,

$$\begin{aligned}\frac{d\alpha_r^{DB}}{da_p} &= (n_r - 1)\frac{d\gamma_{rr}^{DB}}{da_p} + n_p\frac{d\gamma_{rp}^{DB}}{da_p} \\ &= \frac{1}{\left|H^r_{\left(\gamma_{rr},\gamma_{rp}\right)}\right|}\left[-(n_r-1)\left|H^r_{\left(a_p,\gamma_{rp}\right)}\right| + n_p\left|H^r_{\left(a_p,\gamma_{rr}\right)}\right|\right].\end{aligned}$$

Let us first analyze $d\gamma_{rp}^{DB}/da_p$. Since $d\Omega_{rr}/da_p = 0$, by equation 34,

$$\frac{d\gamma_{rp}^{DB}}{da_p} < 0 \iff \left|H^r_{\left(a_p,\gamma_{rr}\right)}\right| < 0 \iff 0 - \frac{d\Omega_{rr}}{d\gamma_{rr}}\frac{d\Omega_{rp}}{da_p} < 0,$$

which is always satisfied. Indeed, by the second order condition on γ_{rr} we have that $\partial\Omega_{rr}/\partial\gamma_{rr} < 0$, and

$$\frac{d\Omega_{rp}}{da_p} = -\gamma_{rp}\left(\alpha_r + \gamma_{rp}\epsilon_r\right) < 0.$$

Second, let us analyze $d\gamma_{rr}^{DB}/da_p$.

$$\frac{d\gamma_{rr}^{DB}}{da_p} = \frac{-\left|H^r_{\left(a_p,\gamma_{rp}\right)}\right|}{\left|H^r_{\left(\gamma_{rr},\gamma_{rp}\right)}\right|},$$

where, using $d\Omega_{rr}/da_p = 0$, and $\frac{d\Omega_{rp}}{da_p} = -\gamma_{rp}\left(\alpha_r + \gamma_{rp}\epsilon_r\right)$,

$$-\left|H^r_{\left(a_p,\gamma_{rp}\right)}\right| = \gamma_{rp}\left(\alpha_r + \gamma_{rp}\epsilon_r\right)\frac{d\Omega_{rr}}{d\gamma_{rp}},$$

and

$$\begin{aligned}\frac{d\Omega_{rr}}{d\gamma_{rp}} &= \frac{\partial\Omega_{rr}}{\partial\alpha_r}\frac{\partial\alpha_r}{\partial\gamma_{rp}} = \frac{\partial\Omega_{rr}}{\partial\alpha_r}n_p \\ \frac{\partial\Omega_{rr}}{\partial\alpha_r} &= (1-2\alpha_r) - \gamma_{rr}\left(1 + \gamma_{rr}\frac{\partial\epsilon_r}{\partial\alpha_r}\right).\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{d\gamma_{rr}^{DB}}{da_p} &< 0 \iff 1 - 2\alpha_r > \gamma_{rr}\left(1 + \gamma_{rr}\frac{\partial\epsilon_r}{\partial\alpha_r}\right) \\ &\iff 1 - 2\alpha_r - \gamma_{rr} > \gamma_{rr}^2\frac{\partial\epsilon_r}{\partial\alpha_r}.\end{aligned}$$ [24]

Finally, let us compute the total effect on α_r^{DB}.

As shown above,

$$\frac{d\alpha_r^{DB}}{da_p} = \frac{1}{\left|H^r_{\left(\gamma_{rr},\gamma_{rp}\right)}\right|}\left[-(n_r-1)\left|H^r_{\left(a_p,\gamma_{rp}\right)}\right| + n_p\left|H^r_{\left(a_p,\gamma_{rr}\right)}\right|\right].$$

Therefore,

$$\begin{aligned}
\frac{d\alpha_r}{da_p} &< 0 \iff -(n_r-1)\left|H^r_{(a_p,\gamma_{rp})}\right| + n_p\left|H^r_{(a_p,\gamma_{rr})}\right| < 0 \\
&\iff -(n_r-1)\left(\frac{d\Omega_{rr}}{da_p}\frac{d\Omega_{rp}}{d\gamma_{rp}} - \frac{d\Omega_{rp}}{da_p}\frac{d\Omega_{rr}}{d\gamma_{rp}}\right) + n_p\left(\frac{d\Omega_{rr}}{da_p}\frac{d\Omega_{rp}}{d\gamma_{rr}} - \frac{d\Omega_{rr}}{d\gamma_{rr}}\frac{d\Omega_{rp}}{da_p}\right) < 0. \quad (42) \\
\frac{d\alpha_r}{da_p} &< 0 \iff \frac{d\Omega_{rp}}{da_p}\left[(n_r-1)\frac{d\Omega_{rr}}{d\gamma_{rp}} - n_p\frac{d\Omega_{rr}}{d\gamma_{rr}}\right] < 0.
\end{aligned}$$

We have seen from Point 1 that $d\Omega_{rr}/da_p = 0$ and $\frac{d\Omega_{rp}}{da_p} < 0$. Using these results, one can rewrite the condition as

$$\frac{d\alpha_r}{da_p} < 0 \iff (n_r-1)\left(\frac{d\Omega_{rr}}{d\gamma_{rp}}\right) - n_p\left(\frac{d\Omega_{rr}}{d\gamma_{rr}}\right) > 0,$$

where, $\frac{d\Omega_{rr}}{d\gamma_{rp}} = \frac{\partial\Omega_{rr}}{\partial\alpha_r}\frac{\partial\alpha_r}{\partial\gamma_{rp}} = \frac{\partial\Omega_{rr}}{\partial\alpha_r}n_p$ and $\frac{d\Omega_{rr}}{d\gamma_{rr}} = \frac{\partial\Omega_{rr}}{\partial\gamma_{rr}} + (n_r-1)\frac{\partial\Omega_{rr}}{\partial\alpha_r}$ from (34) so that

$$\frac{d\alpha_r}{da_p} < 0 \iff -n_p\frac{\partial\Omega_{rr}}{\partial\gamma_{rr}} > 0,$$

which is always true since $\frac{\partial\Omega_{rr}}{\partial\gamma_{rr}} = -(\alpha_r + \gamma_{rr}\epsilon_r) < 0$.

13.2 Comparative statics of α_p^{DB}

Using the budget constraint, we know that

$$\frac{d\alpha_p^{DB}}{da_p} = (n_p-1)\frac{d\gamma_{pp}^{DB}}{da_p} + n_r\frac{d\gamma_{pr}^{DB}}{da_p},$$

where

$$\frac{d\gamma_{pr}^{DB}}{da_p} = \frac{\left|H^p_{(a_p,\gamma_{pp})}\right|}{-\left|H^p_{(\gamma_{pr},\gamma_{pp})}\right|} = \frac{\left|H^p_{(a_p,\gamma_{pp})}\right|}{\left|H^p_{(\gamma_{pp},\gamma_{pr})}\right|}.$$

Therefore,

$$\begin{aligned}
\frac{d\alpha_p^{DB}}{da_p} &= \frac{1}{\left|H^p_{(\gamma_{pp},\gamma_{pr})}\right|}\left[-(n_p-1)\left|H^p_{(a_p,\gamma_{pr})}\right| + n_r\left|H^p_{(a_p,\gamma_{pp})}\right|\right] < 0 \\
&\iff (n_p-1)\left(\frac{d\Omega_{pp}}{da_p}\frac{d\Omega_{pr}}{d\gamma_{pr}} - \frac{d\Omega_{pr}}{da_p}\frac{d\Omega_{pp}}{d\gamma_{pr}}\right) + n_r\left(\frac{d\Omega_{pp}}{d\gamma_{pp}}\frac{d\Omega_{pr}}{da_p} - \frac{d\Omega_{pp}}{d\gamma_{pr}}\frac{d\Omega_{pp}}{da_p}\right) > 0, \quad (43)
\end{aligned}$$

where

$$\frac{d\Omega_{pp}}{d\gamma_{pp}} = \frac{\partial\Omega_{pp}}{\partial\gamma_{pp}} + \frac{\partial\Omega_{pp}}{\partial\alpha_p}\frac{\partial\alpha_p}{\partial\gamma_{pp}}\frac{\partial\alpha_p}{\partial\gamma_{pr}}\left(\frac{\partial\alpha_p}{\partial\gamma_{pr}}\right)^{-1},$$

with

$$\frac{\partial\Omega_{pp}}{\partial\alpha_p}\frac{\partial\alpha_p}{\partial\gamma_{pr}} = \frac{d\Omega_{pp}}{d\gamma_{pr}}.$$

Hence, we can rewrite $d\Omega_{pp}/d\gamma_{pp}$ as

$$\frac{d\Omega_{pp}}{d\gamma_{pp}} = \frac{\partial\Omega_{pp}}{\partial\gamma_{pp}} + \frac{d\Omega_{pp}}{d\gamma_{pr}}\frac{n_p-1}{n_r}.$$

Similarly,

$$\frac{d\Omega_{pr}}{d\gamma_{pr}} = \frac{\partial\Omega_{pr}}{\partial\gamma_{pr}} + \frac{\partial\Omega_{pr}}{\partial\alpha_p}\frac{\partial\alpha_p}{\partial\gamma_{pr}}\frac{\partial\alpha_p}{\partial\gamma_{pp}}\left(\frac{\partial\alpha_p}{\partial\gamma_{pp}}\right)^{-1},$$

with

$$\frac{\partial\Omega_{pr}}{\partial\alpha_p}\frac{\partial\alpha_p}{\partial\gamma_{pp}} = \frac{d\Omega_{pr}}{d\gamma_{pp}}.$$

Hence, we can rewrite $d\Omega_{pr}/d\gamma_{pr}$ as

$$\frac{d\Omega_{pr}}{d\gamma_{pr}} = \frac{\partial\Omega_{pr}}{\partial\gamma_{pr}} + \frac{d\Omega_{pr}}{d\gamma_{pp}}\frac{n_r}{n_p - 1}.$$

Condition (43) can be rewritten as

$$\frac{d\alpha_p}{da_p} < 0 \iff \frac{d\Omega_{pp}}{da_p}\left[(n_p - 1)\frac{d\Omega_{pr}}{d\gamma_{pr}} - n_r\frac{d\Omega_{pp}}{d\gamma_{pr}}\right] - \frac{d\Omega_{pr}}{da_p}\left[(n_p - 1)\frac{d\Omega_{pp}}{d\gamma_{pr}} - n_r\frac{d\Omega_{pp}}{d\gamma_{pp}}\right] > 0.$$

Substituting for $d\Omega_{pp}/d\gamma_{pp}$ and $d\Omega_{pr}/d\gamma_{pr}$, one obtains after simplification,

$$\frac{d\alpha_p}{da_p} < 0 \iff \frac{d\Omega_{pp}}{da_p}\left[(n_p - 1)\frac{\partial\Omega_{pr}}{\partial\gamma_{pr}} + n_r\left(\frac{d\Omega_{pr}}{d\gamma_{pp}} - \frac{d\Omega_{pp}}{d\gamma_{pr}}\right)\right] + n_r\frac{d\Omega_{pr}}{da_p}\frac{\partial\Omega_{pp}}{\partial\gamma_{pp}} > 0.$$

Rearranging,

$$\frac{d\alpha_p}{da_p} < 0 \iff (n_p - 1)\frac{d\Omega_{pp}}{da_p}\left(-\frac{\partial\Omega_{pr}}{\partial\gamma_{pr}}\right) + n_r\frac{d\Omega_{pr}}{da_p}\left(-\frac{\partial\Omega_{pp}}{\partial\gamma_{pp}}\right) + n_r\frac{d\Omega_{pp}}{da_p}\left(\frac{d\Omega_{pp}}{d\gamma_{pr}} - \frac{d\Omega_{pr}}{d\gamma_{pp}}\right) < 0,$$

where

$$\begin{aligned}
\frac{\partial\Omega_{pr}}{\partial\gamma_{pr}} &= -a_r\left(\alpha_p + \gamma_{pr}\epsilon_p\right) < 0,\\
\frac{\partial\Omega_{pp}}{\partial\gamma_{pp}} &= -\left(\alpha_p + \gamma_{pp}\epsilon_p\right) < 0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2\pi_{pp}}{\partial\gamma_{pp}\partial a_p} &< 0 \iff \frac{d\Omega_{pp}}{da_p} = -\gamma_{pp}^2\frac{\partial\epsilon_p}{\partial a_p} < 0,\\
\frac{\partial^2\pi_{pr}}{\partial\gamma_{pr}\partial a_p} &< 0 \iff \frac{d\Omega_{pr}}{da_p} = (1 - \alpha_p)\,\alpha_p - a_r\gamma_{pr}^2\frac{\partial\epsilon_p}{\partial a_p} < 0.
\end{aligned}$$

CEPS
INSTEAD

3, avenue de la Fonte
L-4364 Esch-sur-Alzette
Tél.: +352 58.58.55-801
www.ceps.lu